# Performance Analysis of BPSK, QPSK and TQAM-16 Using the MGF Approach over $\alpha$-$\eta$-$\mu$ Fading Channel

Jyoti Gupta* and Ashish Goel

Jaypee Institute of Information Technology, Noida, Uttar Pradesh, India
E-mail: jyotiguptas21@gmail.com (J.G.); ashish.goel@jiit.ac.in (A.G.)
*Corresponding author

***Abstract*—In this paper, the authors propose a novel, precise, and approximate expression for the generalized fading model using the Moment-Generating Function (MGF). We have evaluateda closed-form mathematical solution for the Average Bit Error Rate (ABER) of Binary Phase Shift Keying (BPSK), the Symbol Error Rate (SER)or symbol error probability (SEP) of Quadrature Phase Shift Keying (QPSK), andTriangular Quadrature Amplitude Modulation (TQAM) over$\alpha-\eta-\mu$ fading channel. Notably, this fading model represents a small-scale variation of the fading signal and encompasses important fading models like Rayleigh, Nakagami-m, Hoyt, one-sided Gaussian, Weibull, and η-μ as special cases. Additionally, it accurately captures the nonlinearity and non-homogeneous nature of the fading channel without a Line-of-Sight (LOS) component. To simplify the evaluation of ABER and SER, we utilize exponential-based approximations of the Gaussian $Q$-function, providing an accurate and mathematically straightforward solution to the SER integral. This approach simplifies the complicated integrals, resulting in an analytically tractable SER expression. The SER results of QPSK, obtained through the exact, proposed analytical expression, Monte Carlo simulation methods, and, result obtained using the $Q$-function approximation proposed by M. Bilim and D. Karaboga for the combination of fading parameters ($a = 0.5$, $\eta = 1.2$, $\mu = 1.5$) are 0.0163445278046601, 0.0163403059168915, 0.0162660000000000, and 0.0186945839664040, respectively, at 28 dB. These findings affirm the superiority of the proposed scheme. We have validated the analytical findings through Monte-Carlo simulations. Moreover, the analytical and simulated SER curves presented in this paper for various modulation formats and fading parameter values further confirm the effectiveness of the proposed SER expression.***

***Keywords*—Average Bit Error Rate (ABER), Symbol Error Rate (SER), Moment-Generating Function (MGF), α-η-μ distribution and Gaussian $Q$-function.**

## I. INTRODUCTION

In the modern era, technology is indispensable, and cell phones are widely used. As a result of its numerous applications, wireless communication continues to catch researcher's attention. In a wireless communication system, ensuring reliable reception of signals and hence achieving a low error rate is of utmost importance. These parameters are essential for designing robust communication systems that enable error-free bit reception at the receiver, maximizing data rates, and optimizing the communication channels [1]. Here, we address this issue by finding out the closed form solution of error performance for various modulation schemes. There are various types of fading channels. G. Fraidenraich and M. D. Yacoub [2] introduced two generalized fading distributions, the $\alpha-\eta-\mu$ and $\alpha-\kappa-\mu$ .In the presented work, the authors chose the generalized fading channel $\alpha-\eta-\mu$. In this paper, the work is devoted to computing average BER, and SER over the $\alpha-\eta-\mu$ fading channel, which helps in evaluating other important fading channels as their special cases like Nakagami-m, Rayleigh, Nakagami-$q$ (Hoyt), $\eta-\mu$ , Weibull, and one-sided Gaussian.

In the performance evolution of wireless transmission with numerous potential system configurations, MGF-based methodology is one of the best ways to estimate the bit error rate [3]. This work aims to generate unified MGF expressions for $\alpha-\eta-\mu$generalized model. The formulas are illustrated using straightforward mathematical operations. Essential performance parameters, including the average bit error rate, can be easily, directly, and unrestrictedly evaluated using the generated MGF expression [3]. BER, and SER are important metrics of numerous modulation techniques over Additive White Gaussian Noise (AWGN) and fading channels in communication systems.

When analyzing the performance of wireless communication systems over AWGN and fading channels, the exact form of the Gaussian $Q$-function is essential. Nevertheless, this form leads to an impracticable definite integral when attempting to compute important metrics such as the Symbol Error Rate (SER) of various digital modulation methods over a fading channel. Therefore, it is essential to transform the exact expression of the Gaussian

Manuscript received February 1, 2024; revised March 18, 2024; accepted April 11, 2024; published August 13, 2024.

$Q$-function into more manageable forms. There are a number of Gaussian $Q$-function approximations and bounds available in the literature [4–16] to satisfy this criteria. In [16], we compared available $Q$-function approximations [4–15] with [16] and demonstrated that our approximation performs the best. The proposed approximation [16] is a novel, straightforward, and tighter approximation of the $Q$-function that makes use of the two-point Gauss Quadrature rule, considering its validity for up to n points. The selection of nodes and weights for their evaluation was carefully made to minimize the approximation error. Therefore, to achieve better accuracy of error performance over fading channels, we have chosen the approximation of $Q$-function proposed in [16].

Furthermore, a closed-form solution is required to analyze the effect of fading in wireless communication. In this context, we have derived a novel exact and approximate expression for α-η-μ fading using the MGF approach. We propose a closed-form solution for theAverage Bit Error Rate (ABER) of BPSK, the Symbol Error Rate (SER) of QPSK, and TQAM-16 modulation.

This paper is divided into four sections. In Section II, the generalized models under consideration are thoroughly reviewed. We develop new, unified Moment Generating Function (MGF) formulas for generalized wireless fading distributions. The applications that show how the newly developed MGF expressions can be used to examine wireless communication networks are presented in Section III. We use these MGF formulas to derive average bit and symbol error rates and compare them to numerically computed results and discoveries from the literature. The contributions presented in this work are finally concluded in Section IV.

## II. α-η-μ Fading Channel and Error Performance Analysis

### A. The α-η-μ Distribution

The pdf of generalized α-η-μ distribution is [17].

$$f_\gamma(\gamma) = \left[\frac{\sqrt{\pi}\,\mu^{\mu+0.5}\,\alpha(\eta-1)^{0.5-\mu}(\eta+1)^{0.5+\mu}}{2\sqrt{\eta}\,\Gamma(\mu)}\right]\times$$

$$\left[\frac{\gamma}{\bar{\gamma}}\right]^{0.5\alpha(\mu+0.5)-1} e^{-\left[\frac{\mu(1+\eta)^2}{2\eta}\right]\left[\frac{\gamma}{\bar{\gamma}}\right]^{0.5\alpha}} I_{\mu-0.5}\left(\frac{(\eta^2-1)\mu}{2\eta}\left[\frac{\gamma}{\bar{\gamma}}\right]^{0.5\alpha}\right) \quad (1)$$

where $\alpha$, $\eta$ and $\mu$ represent the factors responsible for dealing with nonlinearity, dealing with unequal power distribution between in-phase and quadrature components, and taking into account the number of multipath clusters, respectively [17]. Here, $\bar{\gamma}$ defines the average SNR, $\gamma$ represents the instantaneous Signal-to-Noise Ratio (SNR),$I_v(\cdot)$corresponds to the modified Bessel function of the first kind, and $\Gamma(\cdot)$denotes the Euler gamma function.

### B. Generalized Average Bit Error Rate of BPSK

The average symbol or bit error rate for binary symbols affected by a fading channel can be computed by averaging the symbol error rate over the AWGN channel, utilizing the probability density function of the fading envelope [17]. Consequently, the resulting formula obtained through this averaging procedure typically incorporates the $Q$-functionwithin the equation[17]:

$$P_e = \int_0^\infty Q\left(\sqrt{2\gamma}\right) f_\gamma(\gamma) d\gamma \quad (2)$$

where $Q(\cdot)$is the $Q$-function represented as [1]:

$$Q(x) = \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}}\right) \quad (3)$$

Using recentlyproposed approximation of $erfc(x)$ for $n = 3$[16], we can write

$$erfc(x) \approx \frac{1}{3}\,[e^{-44.1103x^2}\cosh(37.9011x^2) + e^{-2.1945x^2}\cosh(0.6533x^2) + e^{-1.1022x^2}\cosh\,(0.0898x^2)] \quad (4)$$

### C. Generalized Moment Generating Function (MGF)

The Laplace Transform of the fading PDF can be used to determine the Moment Generating Function (MGF) [17]. The moment-generating function can be expressed in terms of theLaplace Transform.

$$M(s) = \int_0^\infty f_\gamma(\gamma) e^{-s\gamma}\, d\gamma \quad (5)$$

Eq. (1) can also be represented as:

$$f_\gamma(\gamma) = H\gamma^{L-1} e^{-q\gamma^{\bar{\alpha}}} I_v(P\gamma^{\bar{\alpha}}) \quad (6)$$

where $I_v(.)$ is thefirst kind modified Bessel function.

Using Eqs. (5) and (6), we get

$$M(s) = H\int_0^\infty e^{-s\gamma}\,\gamma^{L-1} e^{-q\gamma^{\bar{\alpha}}} I_v(P\gamma^{\bar{\alpha}})\mathrm{d}\gamma \quad (7)$$

According to [18] and [19],

$$I_v(z) = \sum_{j=0}^{\infty} \frac{\left(\frac{z}{2}\right)^{2j+v}}{j!\,\Gamma(j+v+1)} \quad (8)$$

$$e^{-\tilde{\beta}z} = G_{0,1}^{1,0}\left(\tilde{\beta}z\middle|{}_{0}^{-}\right) \quad (9)$$

With a few manipulations, Eq. (7) can be expressed as

$$M(s) = \frac{H}{\bar{\alpha}P^{L/\bar{\alpha}}}\int_0^\infty e^{-\tilde{s}z^{2/\alpha}}\, z^{\frac{L}{\bar{\alpha}}-1} e^{-\tilde{\beta}z} I_v(z)\mathrm{d}z \quad (10)$$

where$\tilde{s} = \frac{s}{P^{1/\bar{\alpha}}}$and $\tilde{\beta} = q/P$

Eq. (10) yields the expression by combining terms from Eqs. (8) and (9)

$$M(s) = \sum_{i=0}^{\infty} G\int_0^\infty z^{X-1}\, G_{0,1}^{1,0}\left(\tilde{s}z^{2/\alpha}\middle|{}_{0}^{-}\right) G_{0,1}^{1,0}\left(\tilde{\beta}z\middle|{}_{0}^{-}\right) \quad (11)$$

where $G = \frac{H}{i!\,2^{2i+v}\,\Gamma(i+v+1)\bar{\alpha}P^{L/\bar{\alpha}}}$and$X = \frac{L}{\bar{\alpha}} + 2i + v$

Using [19, eqn. (2.24.1.1)], a closed-form solution to the integral in Eq. (11) can found, and is given by

$$M(s) = \sum_{i=0}^{\infty} G\left[\frac{k^{0.5}l^{0.5+X-1}}{\tilde{\beta}^X(2\pi)^{0.5(l+k)-1}}\right] G_{l,k}^{k,l}\left(\frac{l^l}{k^k\tilde{\beta}^l}\tilde{s}^k\middle|{}_{\phi(k,0)}^{\phi(l,1-X)}\right) (12)$$

where $\phi(k,h) = \frac{h}{k}, \frac{h+1}{k}, \cdots, \frac{h+k-1}{k}$ , and here, we specified $2/\alpha = l/k$ such that the $\gcd(l,k) = 1$ (greatest common divisor is 1) to account for non-integer values of $\alpha$.

where,

$$H = \frac{0.5\alpha\sqrt{\pi}\mu^{\mu+0.5}\,(\eta-1)^{0.5-\mu}\,(\eta+1)^{0.5+\mu}}{\sqrt{\eta}\,\Gamma(\mu)\bar{\gamma}^{[0.5\alpha(\mu+0.5)]}}, q = \frac{\mu(1+\eta)^2}{2\eta\bar{\gamma}^{0.5\alpha}},$$

$$L = 0.5\alpha(\mu+0.5), \bar{\alpha} = 0.5\,\alpha, v = \mu - 0.5,$$

$$P = \frac{0.5\,(\eta^2-1)\mu}{\eta\bar{\gamma}^{0.5\alpha}}$$

### *D. The Average Bit Error Rate of BPSK over α-η-μ Fading Channel*

The average bit error rate of BPSK over α-η-μ fading channel Eq. (2) can also be written into the following form using Eq. (3)

$$P_e = \frac{1}{2}\int_0^\infty erfc(\sqrt{\gamma})\, f_\gamma(\gamma)d\gamma \tag{13}$$

Now using approximation of $erfc(x)$given in Eq. (4), we can write Eq. (13) as

$$P_e = \frac{1}{12}\int_0^\infty [\textstyle\sum_{r=1}^{6} e^{-R_m\gamma}]\, f_\gamma(\gamma)d\gamma \tag{14}$$

Now using Eq. (5), we can write Eq. (14) as

$$P_e = \frac{1}{12}[\textstyle\sum_{m=1}^{6} M(R_m)] \tag{15}$$

where,
$[R_m]_{m=1}^{6} = [6.2092, 82.0116, 1.5412, 2.848, 1.0124, 1.192]$

### *E. SER of QPSK over α-η-μ Fading Channel*

The exact expression of symbol error rate for QPSK over the AWGN channel is defined as [20]:

$$SER_{AWGN} = 2\,Q(\sqrt{\gamma}) - Q^2(\sqrt{\gamma}) \tag{16}$$

As per the definition, the symbol error rate of QPSK over the α-η-μ fading channel can be calculated using following expression:

$$SER_{fading} = \int_0^\infty SER_{AWGN}\, f_\gamma(\gamma)d\gamma \tag{17}$$

By employing Eq. (16), we can express Eq. (17) in the following form:

$$SER_{fading} = \int_0^\infty \{2\,Q(\sqrt{\gamma}) - Q^2(\sqrt{\gamma})\}.f_\gamma(\gamma)d\gamma \tag{18}$$

Using the approximation of $erfc(x)$ given in Eq. (4) to Eq. (18), we can now write the final SER expression of QPSK over $\alpha-\eta-\mu$ fading, as

$$SER_{fading} = \frac{1}{6}[\textstyle\sum_{t=1}^{6} M(\varkappa1_t)] - \frac{1}{144}[\textstyle\sum_{t=1}^{6} M(\varkappa2_t) + \{2\,\textstyle\sum_{t=1}^{15} M(\varkappa3_t)\}] \tag{19}$$

where,

$$[\varkappa1_t]_{t=1}^{6} = [3.1046, 41.0058, 0.7706, 1.424, 0.5062, 0.596]$$

$$[\varkappa2_t]_{t=1}^{6} = [6.2092, 82.0116, 1.5412, 2.848, 1.0124, 1.192]$$

$$[\varkappa3_t]_{t=1}^{15} = \begin{bmatrix} 44.1104, 3.8752, 4.5286, 3.6108, 3.7006, \\ 41.7764, 42.4298, 41.512, 41.6018, 2.1946, \\ 1.2768, 1.3666, 1.9302, 2.02, 1.1022 \end{bmatrix}$$

### *F. SER of TQAM-16 over α-η-μ Fading Channel*

According to [21], the standardized equation of SER for different modulation techniques over the AWGN channel is as follows:

$$P_{AWGN} = KQ(\sqrt{\varsigma\gamma}) + \frac{2}{3}K_C Q^2\left(\sqrt{\frac{2\varsigma\gamma}{3}}\right) - 2K_C Q(\sqrt{\varsigma\gamma})Q\left(\sqrt{\frac{\varsigma\gamma}{3}}\right) \tag{20}$$

The Signal-to-Noise Ratio (SNR) in this case is represented by $\gamma$, while the modulation technique parameters, average count of nearest-neighbours, and average count of pairs of adjacent nearest-neighbours are represented by $\varsigma$, $K$, and $K_C$, respectively [21]. The following $\varsigma = 2/9$, $K = 33/8$ and $K_C = 27/8$ are the defined SER parameters for TQAM-16 constellations [21]. The symbol error rate of any digital modulation technique is typically specified by the linear combinations of integrals below, or their special cases [1]:

$$I_1 = \int_0^\infty Q(A\sqrt{\gamma})\,Q(B\sqrt{\gamma})f_\gamma(\gamma)d\gamma \tag{21a}$$

$$I_2 = \int_0^\infty Q^\delta\,(A\sqrt{\gamma})f_\gamma(\gamma)d\gamma \tag{21b}$$

where, $\delta$ is the order of $Q(.)$ and $f_\gamma(\gamma)$ is the probability density function of the channel. $A$ and $B$ are the real positive constants that vary depending on the specific digital modulation scheme [1].

As per the definition, the SER of TQAM-16 over fading channels can be derived as:

$$SER_{fading} = P_{fading} = \int_0^\infty P_{AWGN}.f_\gamma(\gamma)d\gamma \tag{22}$$

By employing both Eq. (20) and Eq. (22), we can derive a novel equation for SER over$\alpha-\eta-\mu$fading channel for TQAM-16.

$$SER_{fading} = \int_0^\infty \left\{\frac{33}{8}Q\left(\sqrt{\frac{2}{9}\gamma}\right) + \frac{9}{4}Q^2\left(\sqrt{\frac{4}{27}\gamma}\right) - \frac{27}{4}Q\left(\sqrt{\frac{2}{9}\gamma}\right)Q\left(\sqrt{\frac{2}{27}\gamma}\right)\right\}.f_\gamma(\gamma)d\gamma \tag{23}$$

The final expression of symbol error rate over $\alpha-\eta-\mu$ fading channel for TQAM-16 is obtained by applying the complementary error function $erfc(x)$ approximation Eq. (4), in Eq. (23).

$$SER_{fading} = \frac{33}{96}[\textstyle\sum_{i=1}^{6} M(W1_i)] + \frac{1}{64}[\textstyle\sum_{i=1}^{6} M(W2_i) + \{2\,\textstyle\sum_{i=1}^{15} M(W3_i)\}] - \frac{3}{64}[\textstyle\sum_{i=1}^{36} M(W4_i)] \tag{24}$$

where,

$$[W1_i]_{i=1}^{6} = [0.6899, 9.1124, 0.1712, 0.3164, 0.1125, 0.1324]$$

$$[W2_i]_{i=1}^{6} = [0.9198, 12.1498, 0.2284, 0.422, 0.15, 0.1776]$$

$$[W3_i]_{i=1}^{15} = \begin{bmatrix} 6.5348, 0.5741, 0.6709, 0.5349, 0.5482, \\ 6.1891, 6.2859, 6.1499, 6.1632, 0.3252, \\ 0.1892, 0.2025, 0.286, 0.2993, 0.1633 \end{bmatrix};$$

$$[W4_i]_{i=1}^{36} = \begin{bmatrix} 0.9199, 9.3424, 0.4012, 0.5464, 0.3425, 0.3624, \\ 3.7274, 12.1499, 3.2087, 3.3539, 3.15, 3.1699, \\ 0.747, 9.1695, 0.2283, 0.3735, 0.1696, 0.1895, \\ 0.7954, 9.2179, 0.2767, 0.4219, 0.218, 0.2379, \\ 0.7274, 9.1499, 0.2087, 0.3539, 0.15, 0.1699, \\ 0.734, 9.1565, 0.2153, 0.3605, 0.1566, 0.1765 \end{bmatrix}$$

## III. Results and Discussion

The accuracy of the proposed closed approximation is tested for different values of fading parameters. As a result, Fig.1 depicts that the average bit error rate graph obtained through the approximation aligns with the exact results, the analytical results, and the outcomes from Monte Carlo simulations across all possible cases of the $\alpha-\eta-\mu$ fading channel. Additionally, it is noticeable from Fig. 1 that the ABER performance of the BPSK modulation technique over the fading channel for $\alpha = 0.5,\ \eta = 1.2$ and $\mu = 2$ is better than that for $\alpha = 0.5,\ \eta = 1.2$ and $\mu = 1$and it is further improved when $\alpha = 1.5,\ \eta = 1.7$ and $\mu = 1$. From these trends in ABER results, we can affirm that, for a given value of α, either κ or μ, or increasing both of them, enhances the ABER performance of the BPSK modulation technique. The ABER degrades as we increase the fading parameter $\alpha$.

Additionally, Table I compares the ABER of BPSK over $\alpha-\eta-\mu$fading channel in terms of accuracy. Table I highlights that the exact value of average BER of BPSK for $\alpha = 0.5, \eta = 1.2$ and $\mu = 1$, is 0.0952331636163539 at SNR 10 dB and their corresponding analytical and simulation results are 0.0952652193693010 and 0.0954500000000000, respectively. Similarly, the actual, analytical, and simulation values of the average BER of BPSK at 10 dB are 0.00311760605032327, 0.00311692808400305, and 0.00313175000000000, respectively, for $\alpha = 1.5, \eta = 1.7$ and $\mu = 2$ . These findings demonstrate that, for all SNR values, the results obtained by analytical expression, Monte Carlo simulations, and exact expression using $Q$-function are also virtually indistinguishable.

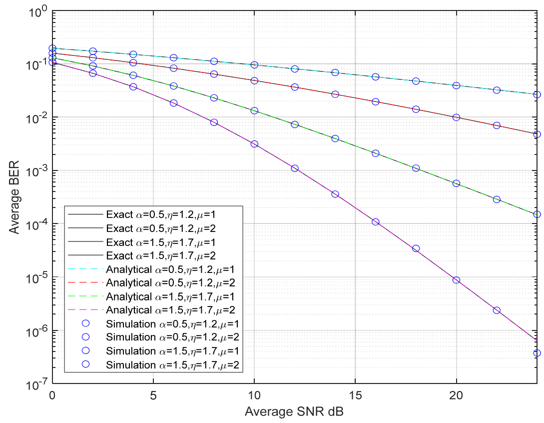


Fig. 1. Average BER of BPSK over $\alpha-\eta-\mu$ fading channel.

TABLE I. Accuracy Comparison OF Average BER of BPSK over $\alpha-\eta-\mu$ Fading Channel

| | BPSK | | |
|---|---|---|---|
| SNR (dB) | Exact | Analytical | Simulation |
| $\alpha = 0.5, \eta = 1.2, \mu = 1$ | | | |
| 0 | 0.195571254757108 | 0.195741445724338 | 0.195900000000000 |
| 10 | 0.0952331636163539 | 0.0952652193693010 | 0.0954500000000000 |
| 20 | 0.0393130079327208 | 0.0393183147814793 | 0.0390140000000000 |
| $\alpha = 0.5, \eta = 1.2, \mu = 2$ | | | |
| 0 | 0.158696471825131 | 0.158489026352692 | 0.158759000000000 |
| 10 | 0.0486948114754481 | 0.0486631170792050 | 0.0481885000000000 |
| 20 | 0.00991889026230215 | 0.00991581937241815 | 0.00986400000000000 |
| $\alpha = 1.5, \eta = 1.7, \mu = 1$ | | | |
| 0 | 0.129360127023766 | 0.129457723972482 | 0.129259500000000 |
| 10 | 0.0131682902725477 | 0.0131713934833430 | 0.0131803750000000 |
| 20 | 0.000563127132142243 | 0.000563206852516639 | 0.000570250000000000 |
| $\alpha = 1.5, \eta = 1.7, \mu = 2$ | | | |
| 0 | 0.106299299898227 | 0.107574791614436 | 0.106333250000000 |
| 10 | 0.00311760605032327 | 0.00311692808400305 | 0.00313175000000000 |
| 20 | 8.83993040290858e-06 | 8.83890630193674e-06 | 8.75000000000000e-06 |

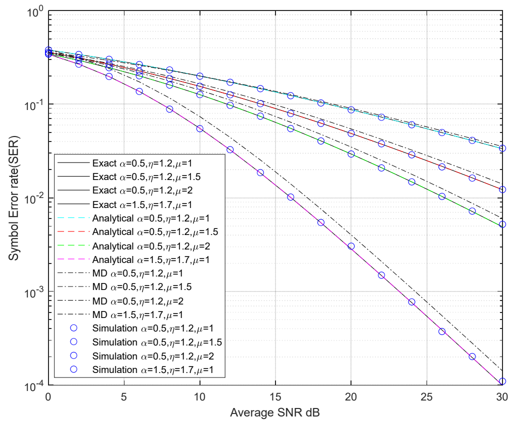


Fig. 2. SER of QPSK over $\alpha-\eta-\mu$ fading channel.

Fig. 2 shows how different fading variables affect the performance of the systems using QPSK modulation, emphasizing how fading parameters affect symbol error rate and making this tendency quite evident: if one or more fading parameters, α, η, or μ, of the channel increases, the SER performance of QPSK modulation scheme decreases. The presented graphs in Fig. 2 show that the computer simulations and the derived mathematical formulation are in full agreement. Furthermore, the $Q$-function approximation put forward by Bilim and Karaboga [15] is also utilized to establish the mathematical expression of SER for QPSK modulation; however, it is found to be less precise than the method derived in this work. The other technique is shown in Fig. 2 as being inaccurate at low SER values. As a result, the proposed closed form expression is more accurate, and the findings of Fig. 2 amply show that the proposed approximation performs better in terms of SER accuracy than the results obtained using the approximation proposed by Bilim and Karaboga [15].

Additionally, Table II offers a precise comparison of QPSK's symbol error rate over the $\alpha-\eta-\mu$ fading channel. Table II shows that the exact SER of QPSK for $\alpha = 0.5$, $\eta = 1.2$, $\mu = 1$ is 0.379353095378426 at SNR 0 dB. The results obtained from the Monte Carlo simulations and analytical expressions are 0.379072000000000 and 0.379556944158262, respectively, while the SER determined by Bilim and Karaboga [15] is 0.359955005453684. Similarly, the exact, proposed analytical expression, computer simulations, and result obtained using the $Q$-function approximation proposed by Bilim and Karaboga [15] for fading parameters $\alpha = 0.5$, $\eta = 1.2$, $\mu = 2$ at 14 dB are 0.0740630792356347, 0.0740423482486165, 0.0745020000000000, and 0.0850723651555381, respectively. These findings demonstrate that the exact and proposed results have better match than the $Q$-function approximation proposed by Bilim and Karaboga [15].

TABLE II. ACCURACY COMPARISON OF SER FOR QPSK MODULATION OVER $\alpha-\eta-\mu$ FADING CHANNEL

| | QPSK | | | |
|---|---|---|---|---|
| SNR (dB) | Exact | Analytical | Simulation | M. Bilim and D. Karaboga |
| $\alpha = 0.5, \eta = 1.2, \mu = 1$ | | | | |
| 0 | 0.379353095378426 | 0.379556944158262 | 0.379072000000000 | 0.359955005453684 |
| 14 | 0.146484230892734 | 0.146495591159360 | 0.146616000000000 | 0.148760527304649 |
| 28 | 0.0408819839862550 | 0.0408811619799807 | 0.0410940000000000 | 0.0429838396391849 |
| $\alpha = 0.5, \eta = 1.2, \mu = 1.5$ | | | | |
| 0 | 0.358223921686449 | 0.358032003770186 | 0.358246000000000 | 0.353565205668677 |
| 14 | 0.100933984303989 | 0.100892567854071 | 0.101362000000000 | 0.109873290660234 |
| 28 | 0.0163445278046601 | 0.0163403059168915 | 0.0162660000000000 | 0.0186945839664040 |
| $\alpha = 0.5, \eta = 1.2, \mu = 2$ | | | | |
| 0 | 0.345350914846178 | 0.344997116167442 | 0.346246000000000 | 0.350611679593865 |
| 14 | 0.0740630792356347 | 0.0740423482486165 | 0.0745020000000000 | 0.0850723651555381 |
| 28 | 0.00715801493375546 | 0.00715770009754221 | 0.00722000000000000 | 0.00874111724816721 |
| $\alpha = 1.5, \eta = 1.7, \mu = 1$ | | | | |
| 0 | 0.342002079443748 | 0.342277352767469 | 0.342898000000000 | 0.380630984371228 |
| 14 | 0.0184308605320168 | 0.0184330035745031 | 0.0185940000000000 | 0.0254021615882278 |
| 28 | 0.000197067264911575 | 0.000197075952748361 | 0.000202000000000000 | 0.000280445263779686 |

Fig. 3 illustrates the impact of TQAM-16 modulation schemes for various fading factors, highlighting the effect of fading parameters on SER. Fig. 3clearly shows a clear trend: the SER of the TQAM-16 modulation scheme drops as any fading parameters α, η, or μ rise. The depicted graphs demonstrate the new, derived mathematical expression and computer simulations to be in perfect accord.

In addition, Table III provides a comprehensive numerical comparison of the exact, analytical, and proposed results for the SER of TQAM-16 over $\alpha-\eta-\mu$ fading channel. According to Table III, the actual value of the SER of TQAM-16 for the conditions of $\alpha = 0.5, \eta = 1.3, \mu = 1$ at 40 dB SNR is 0.0338301253120309, and the results derived using the analytical expression and Monte Carlo simulations are 0.0338195271333271, and 0.0336750000000000. It is clear that the proposed analytical and simulation results are closest to the actual value of SER. Furthermore, similar conclusions can also be drawn from Table III for $\alpha = 1.25, \eta = 1.3, \mu = 1$ at 0 dB, 0.746159396867051, 0.745934928764225, and 0.745934928764225 for exact, analytical, and simulation, respectively. As a result, the proposed work is precise for all values of SNRs.

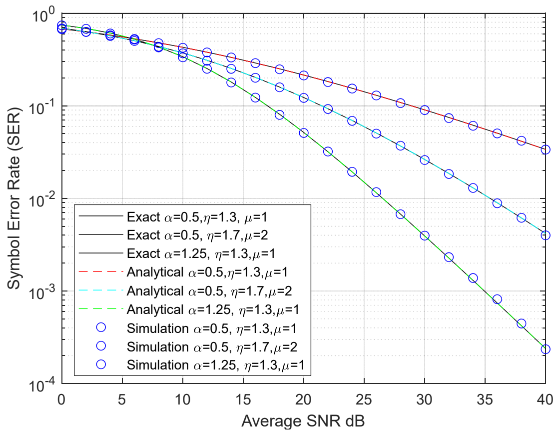


Fig. 3. SER of TQAM-16 over $\alpha-\eta-\mu$ fading channel.

TABLE III. ACCURACY COMPARISON OF SER OF TQAM-16 OVER $\alpha-\eta-\mu$ FADING CHANNEL

| | TQAM-16 | | |
|---|---|---|---|
| SNR (dB) | Exact | Analytical | Simulation |
| $\alpha = 0.5, \eta = 1.3, \mu = 1$ | | | |
| 0 | 0.678635400394207 | 0.676231154643377 | 0.672700000000000 |
| 20 | 0.215160262745535 | 0.214933334240360 | 0.212885000000000 |
| 40 | 0.0338301253120309 | 0.0338195271333271 | 0.0336750000000000 |

| | | | |
|---|---|---|---|
| $\alpha = 0.5, \eta = 1.7, \mu = 2$ | | | |
| 0 | 0.690269985643655 | 0.687617783114101 | 0.685970000000000 |
| 20 | 0.122909639833508 | 0.123097946201206 | 0.121860000000000 |
| 40 | 0.00417701618835439 | 0.00418302883124142 | 0.00399200000000000 |
| $\alpha = 1.25, \eta = 1.3, \mu = 1$ | | | |
| 0 | 0.746159396867051 | 0.745934928764225 | 0.740210000000000 |
| 20 | 0.0515610908459058 | 0.0516211612184090 | 0.0509625000000000 |
| 40 | 0.000238886768744605 | 0.000239045345247564 | 0.000235000000000000 |

## IV. Conclusion

In this paper, the closed-form solutions of average BER, and SER using the Moment-Generating Function (MGF) for BPSK, QPSK, and TQAM-16 over $\alpha-\eta-\mu$ fading channel are derived. The validity of the expression is demonstrated by how closely the exact and simulation results match the analytical results. The proposed closed-form solution of BPSK, QPSK, and TQAM-16 is obtained by using the exponential-based approximations of the Gaussian Q-function. This leads to more efficient computational assessment and streamlined analytical manipulation because of the newly created MGF expressions. The error rates for BPSK, QPSK, and TQAM-16 are analyzed to show the applicability and validity of the new MGF expressions. The proposed results facilitate unrestricted performance analysis and precise planning and inspire researchers to delve further into these emerging fading models.

## Conflict of Interest

The authors declare no conflict of interest.

## Author Contributions

The Mathematical analysis of the paper was done by Ms. Jyoti Gupta and Simulation of work was done by Dr. Ashish Goel. Paper writing and editing work was contributed equally by both of them. All authors approved the final version.

## References


[1] M. K. Simon and M.-S. Alouini, *Digital Communication Over Fading Channels, Hoboken*, NJ, USA: Wiley, 2005.

[2] G. Fraidenraich and M. D. Yacoub, “The α-η-μ and α-κ-μ fading distributions,” in *Proc. IEEE Ninth International Symposium on Spread Spectrum Techniques and Applications*, pp. 16–20, 2006. doi: 10.1109/ISSSTA.2006.311725

[3] E. Salahat, A. Hakam, N. Ali and A. Kulaib, “Moment generating functions of generalized wireless fading channels and applications in wireless communication theory,” in *Proc. 85th Vehicular Technology Conference (VTC Spring)*, Sydney, NSW, Australia, 2017, pp. 1–4. doi: 10.1109/VTCSpring.2017.8108409

[4] J. W. Craig, “A new, simple and exact result for calculating the probability of error for two-dimensional signal constellations,” in *Proc. MILCOM 91 - Conference Record*, McLean, VA, USA, 1991, pp. 571–575. doi: 10.1109/MILCOM.1991.258319

[5] M. Chiani, D. Dardari and M. K. Simon, “New exponential bounds and approximations for the computation of error probability in fading channels,” *IEEE Transactions on Wireless Communications*, vol. 2, no. 4, pp. 840–845, 2003. doi: 10.1109/TWC.2003.814350

[6] G. K. Karagiannidis and A. S. Lioumpas, “An improved approximation for the Gaussian Q-function,” *IEEE Communications Letters*, vol. 11, no. 8, pp. 644–646, 2007. doi: 10.1109/LCOMM.2007.070470

[7] P. Loskot and N. C. Beaulieu, “Prony and polynomial approximations for evaluation of the average probability of error over slow-fading channels,” *IEEE Transactions on Vehicular Technology*, vol. 58, no. 3, pp. 1269–1280, 2009. doi: 10.1109/TVT.2008.926072

[8] Q. Shi and Y. Karasawa, “An accurate and efficient approximation to the Gaussian Q-function and its applications in performance analysis in Nakagami-m fading,” *IEEE Communications Letters*, vol. 15, no. 5, pp. 479–481, 2011. doi: 10.1109/LCOMM.2011.032111.102440

[9] D. Sadhwani, R. N. Yadav and S. Aggarwal, “Tighter bounds on the Gaussian *Q* function and its application in Nakagami-*m* fading channel,” *IEEE Wireless Communications Letters*, vol. 6, no. 5, pp. 574–577, 2017. doi: 10.1109/LWC.2017.2717907

[10] Y. Chen and N. C. Beaulieu, “A simple polynomial approximation to the Gaussian Q-function and its application,” *IEEE Communications Letters*, vol. 13, no. 2, pp. 124–126, 2009. doi: 10.1109/LCOMM.2009.081754

[11] J. Nikolić and Z. Perić, “Novel exponential type approximations of the Q-function,” *Facta Universitatis, Series: Automatic Control and Robotics*, vol. 21, no. 1, pp. 047–058, 2022.

[12] A. Goel and J. Gupta, "Piece-wise approximation for Gaussian Q-function and its applications,” *Sādhanā*, vol. 47, no. 169, 2022. https://doi.org/10.1007/s12046-022-01944-w

[13] Z. Perić, A. Marković, N. Kontrec, J. Nikolić, M. D. Petković, and A. Jovanović, “Two interval upper-bound Q-function approximations with applications,” *Mathematics*, vol. 10, no. 19, 3590, 2022. https://doi.org/10.3390/math10193590

[14] A. Powari, D. Sadhwani, L. Gupta, and R. N. Yadav, “Novel romberg approximation of the Gaussian Q function and its application over versatile κ−μ shadowed fading channel,” *Digital Signal Processing*, vol. 132, no. 103800, 2023.

[15] M. Bilim and D. Karaboga, “Improved Chernoff bound of Gaussian Q-function with ABC algorithm and its QAM applications to DB SC and MRC systems over Beaulieu-Xie channels,” *Physical Communication* vol. 58, no. 102034, 2023.

[16] A. Goel and J. Gupta, “Novel approximation of Q-function using Gauss quadrature rule,” *Sādhanā*, vol. 48, no. 215, 2023. https://doi.org/10.1007/s12046-023-02281-2

[17] E. Salahat and A. Hakam, “Performance analysis of α-η-μ and α-κ-μ generalized mobile fading channels,” in *Proc. 20th European Wireless Conference*, Barcelona, Spain, 2014, pp. 1–6.

[18] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions: With Formulas, Graphs, and Mathematical Tables*. New York: Dover, 1974.

[19] A. P. Prudnikov, Y. A. Brychkov, and O. I. Marichev, *Integrals and Series, Vol. 3: More Special Functions*, Oxford University Press US, 1986.

[20] J G Proakis and M Saleh, *Digital Communications*, 5th ed. McGraw-Hill, New York, 2008.

[21] L. Rugini, “Symbol error probability of hexagonal QAM,” *IEEE Communications Letters*, vol. 20, no. 8, pp. 1523–1526, 2016. doi: 10.1109/LCOMM.2016.2574343