Prime Code efficiency in DS-OCDMA Systems using Parallel Interference Cancellation

C. GOURSAUD-BRUGEAUD, A. JULIEN-VERGONJANNE, J.P. CANCES
XLIM/C^{2}S^2 UMR CNRS 6172, Limoges, France
Email: goursaud@ensil.unilim.fr, anne@ensil.unilim.fr

Abstract—**In this paper, we are studying the Prime Codes (PC) efficiency, in Direct Sequence Optical Code Division Multiple Access system (DS-OCDMA) using Parallel Interference Cancellation receiver (PIC). We develop the analytical expression of the error probability upper bound in the chip synchronous case, for PC. The main result of our work is that the PIC receiver totally suppresses the effect of Multiple Access Interference (MAI) for Prime Codes, and leads to an error free O-CDMA link in the noiseless case, for whichever employed PC. Simulation results are presented to validate the theoretical analysis. We also show that the PIC receiver permits reducing the required code length for a given Bit Error Rate compared to the Conventional Correlation Receiver. Finally, we compare the PC to the Optical Orthogonal Codes (OOC), and show that the use of PC allows a significant reduction of the required SNR.**

Index Terms—**Optical Code Division Multiple Access (OCDMA), Prime Codes, Parallel Interference Cancellation**

I. Introduction

The OCDMA (Optical Code Division Multiple Access) inspired from Radio Frequency communications, is nowadays studied for application in optical networks. This technique is expected to be a multiple access solution for flexible and secure high-speed optical networks especially for high speed LANs [1-2]. Thanks to the robustness of CDMA to multi-path fading, the optical CDMA can also be extended to optical wireless communications [3].

Such a system works by assigning each user a specific code and provides an asynchronous and simultaneous access to several users [1,4].

Coherent and incoherent OCDMA have been investigated, but we focus, in this paper, on incoherent OCDMA systems, which require lower complexity (especially for synchronization) than the coherent ones.

In incoherent systems, unipolar codes are used. As unipolar codes can not be strictly orthogonal, the system suffers from Multiple Access Interference (MAI). In temporal coding systems employing ideal light sources, MAI can be considered as the dominant noise source [5].

One of the simplest ways to reduce MAI consists in using a long spreading code sequence. Nevertheless, the use of long code sequences such as OOC [4] implies ultra short pulses for high data rates, which are difficult to produce. Moreover, the electrical device bandwidth imposes a limitation on the code length. So, there is a tradeoff between number of users, code length and MAI impact. One solution can be the use of short code sequences along with receiver removing MAI.

Prime Codes (PC) [6] permit supporting many simultaneous users with short code length. However, they suffer from high cross-correlation products, which create high amount of MAI. Thus, the system is easier to implement, but MAI has to be mitigated in order to maintain good performance. Several methods have been investigated to reduce MAI effect [7-12], but there still remains an error floor.

We have previously studied in [13-14] the Parallel Interference Cancellation receiver (PIC) [11-12] and shown that it is a performing way of improving the performance of a DS-OCDMA system using Optical Orthogonal Codes OOC.

We have presented in [15] our first promising results on the detection of PC with a PIC receiver. In this paper, we will develop this study more precisely, and confirm the PC interest in comparison with OOC.

We first describe the system. Then, we develop the theoretical expression of the error probability upperbound for the PC. We deduce from this analysis that for the optimal threshold levels, the transmission is error free. Then, we evaluate the benefits of using a PIC instead of a CCR to decode the PC. Finally, we compare the PC to the OOC to highlight their benefits and drawbacks.

II. System description

A. DS-OCDMA system

We consider a incoherent, synchronous Direct Detection OCDMA system.

Each user employs an On/Off Keying (OOK) modulation to transmit independent and equiprobable binary data upon an optical channel.

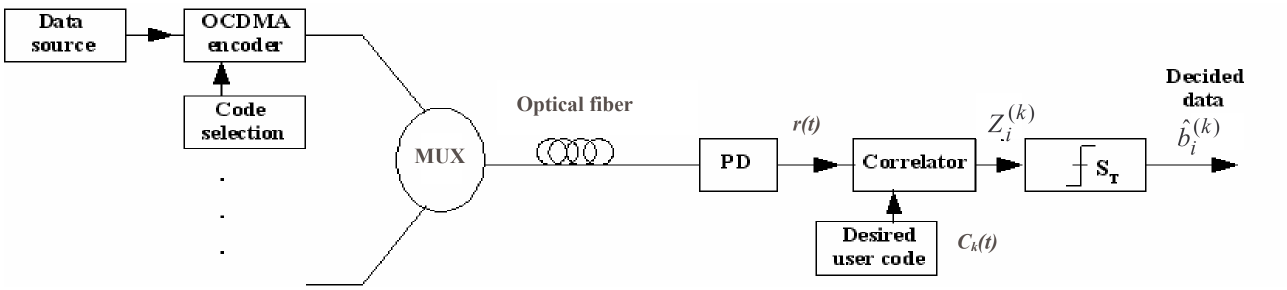

Figure 1. DS-OCDMA system, (PD: photodetector)

A code sequence is impressed upon the binary data by the encoder. The sequence code is specific to each user, in order to be able to extract the data at the end receiver: the received signal would be compared to the sequence code, then to a threshold level at the comparator for the data recovery (fig 1).

B. *Prime Code (PC)*

A unipolar Prime Sequence (PS) [6] of length *P* (*P* being a prime number) is defined by $S_i = \{s_{i,0}, s_{i,1}, ..., s_{i,j}, ..., s_{i,P-1}\}$. Each element $s_{i,j}$ is obtained by $s_{i,j} = i \times j$ (mod *P*), with $i, j \in GF(P)$ the Galois field.

We construct a set of Prime Code (PC) $C_i = \{c_{i,0}, c_{i,1}, ..., c_{i,k}, ..., c_{i,F-1}\}$, whose length is $F = P^2$, from the PS by:

$$c_{i,k} = \begin{cases} 1 & if \;\; k = jP + s_{i,j} \quad (0 \le j < P) \\ 0 & otherwise \end{cases}$$

We obtain *N*=*P* codewords with a length P^2, a weight *W*=*P*, and with crosscorrelation values no greater than 2 . We will refer to such Prime Code with “PC (P^2,*P*)”.

C. *Conventional Correlation Receiver (CCR)*

We first consider in this paper that there is no noise contribution and that the synchronisation is perfect. At the receiver end, $r(t)$ is the sum of the users' signals:

$$r(t) = \sum_{k=1}^{N} b_i^{(k)} C_k(t)$$

with $C_k(t)$ the k^{th} user sequence code, and $b_i^{(k)} \in \{0,1\}$ the i^{th} data bit of the k^{th} user.

As shown in fig 1, the received signal *r(t)* is multiplied by the code sequence corresponding to the desired user $C_k(t)$, and the result is integrated. We get the decision variable value $Z_i^{(k)}$:

$$Z_i^{(k)} = \int_0^T r(t).C_k(t)dt = W.b_i^{(k)} + \sum_{j=1, j\neq k}^{N} b_i^{(j)}.\int_0^T C_k(t).C_j(t)dt \qquad (1)$$

$$= W.b_i^{(k)} + I_{CCRk}$$

with *T* the bit duration.

The term I_{CCRk} is the interference due to all the undesired users (MAI). The decision variable value is compared to the threshold level S_T of the decision device and an estimation of the transmitted bit, $\hat{b}_i^{(k)}$, is given. An error can occur only when $b_i^{(k)}$ is a zero data and the MAI term is greater than the threshold level value S_T.

For our analysis, we consider that the code sequences are PC.

Yang and Kwong [6] have reported the analytical expression of the error probability P_{ECCR} for the ideal chip synchronous case for a threshold value S_T :

$$P_{ECCR} = \frac{1}{2} \cdot \frac{1}{2} \sum_{i_1=0}^{S_T-1} \sum_{i_2=0}^{\lfloor (S_T-1-i_1)/2 \rfloor} \left[\frac{(N-1)!}{i_1! i_2! (N-1-i_1-i_2)!} \left(\frac{p_1}{2}\right)^{i_1} \left(\frac{p_2}{2}\right)^{i_2} \left(1 - \frac{p_1}{2} - \frac{p_2}{2}\right)^{N-1-i_1-i_2} \right] \qquad (2)$$

with p_1, p_2 the average probabilities of having one and two overlaps respectively between 2 codes sequences, defined as:

$$p_1 + 2p_2 = 1 \text{ and } p_2 = \frac{(P+1)(P-2)}{6P^2}$$

D. *Parallel Interference Cancellation receiver (PIC)*

To present the PIC principle, we assume the first user to be the desired one. All the users are supposed to have the same transmitting energy so there is no strongest interfering signal.

The aim of the PIC (fig 2) is to reproduce the interference term due to all interfering users and to remove it from the received signal. The PIC first detects the $N-1$ undesired users employing the

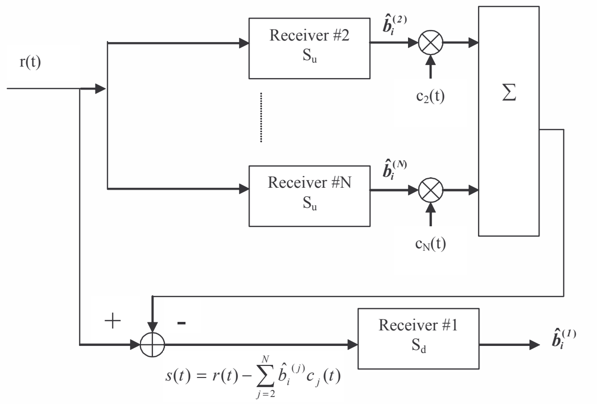

Figure 2. Parallel Interference Cancellation structure

conventional correlation receiver defined in the previous part with a threshold level S_u. The estimated interference is built by spreading the estimated data with the corresponding code sequence. Then, it is removed from the received signal $r(t)$.

The bit sent by the desired user (#1) is evaluated with a conventional correlation receiver with a threshold level S_d. The signal *s(t)* applied to the entry of the receiver is:

$$s(t) = r(t) - \sum_{j=2}^{N} \hat{b}_i^{(j)} C_j(t) = b_i^{(1)} C_1(t) + \sum_{j=2}^{N} (b_i^{(j)} - \hat{b}_i^{(j)}).C_j(t)$$

III. Theoretical Analysis

In this section, the expression of the error probability for the PIC receiver, will be demonstrated in a synchronous case with no noise contribution, for PC (P^2, P), and for N simultaneous users. We consider the same threshold levels S_u $(0 < S_u \le W)$ for the $(N-1)$ undesired users' receivers. We consider that the threshold level for the desired user #1 is S_d $(0 < S_d \le W)$.

Moreover, we consider that all codes have the same overlap probabilities p_1, p_2, although there are few differences. In particular, for the code *C0(t),* p_2 is null. Thus, it can be shown that the user identified by this code is always well detected with a CCR, and so never generates interference on the desired user when using a PIC. As a consequence, in this theoretical analysis, the interference term is overestimated and leads to a theoretical upper bound of the error probability.

In a general way, the error probability can be written as: $P_e = \frac{1}{2} P(\hat{b}_i^{(1)} = 1 / b_i^{(1)} = 0) + \frac{1}{2} P(\hat{b}_i^{(1)} = 0 / b_i^{(1)} = 1)$

The decision-making concerning $\hat{b}_i^{(1)}$ is related to user #1 decision variable, which is expressed as:

$$Z_i^{(1)} = W.b_i^{(1)} + \sum_{j=2}^{N} (b_i^{(j)} - \hat{b}_i^{(j)}) \int_0^T C_1(t).C_j(t)dt$$
$$= W.b_i^{(1)} + \sum_{j=2}^{N} I_i^{(j)} = W.b_i^{(1)} + I \quad (3)$$

I is called the interfering term. Errors on user #1's data are due to this term.

Indeed, for an undesired user #*j*, $I_i^{(j)}$ is the interference term due to user $\#j$ on user #1. $(b_i^{(j)} - \hat{b}_i^{(j)})$ is either null when there is no error, or non null when there is an error. With CCR, an error can occur only when the sent data is a "0", so $(b_i^{(j)} - \hat{b}_i^{(j)}) = -1$. As $\int_0^T C_1(t).C_j(t).dt$ is either null, equal to 1 or equal to 2, $I_i^{(j)}$ is either null, equal to "-1", or to "-2".

When $I_i^{(j)}$ is non-null, the user #j is called "interfering user". The interfering users are the ones that send a '0' detected as a '1', and have at least one common chip with user #1.

As $I_i^{(j)}$ is either null or negative, $I = \sum_{j=2}^{N} I_i^{(j)}$ is always negative or null, so the decision variable $Z_i^{(1)}$ is less (or equal) than it should be. So, there can be errors only for $b_i^{(1)} = 1$. Therefore, the expression of the error probability can be simplified in the noiseless case to:

$$P_e = \frac{1}{2} P(\hat{b}_i^{(1)} = 0 / b_i^{(1)} = 1)$$

From now on, we consider the case $b_i^{(1)} = 1$.

In this case, P_e can be expressed as a function of the probabilities of two events:

- the probability P_{i1} for an undesired user #j who sent "0" to create an interference of "-1" on user #1, when $b_i^{(1)} = 1$,
- the probability P_{i2} for an undesired user #j who sent "0" to create an interference of "-2" on user #1, when $b_i^{(1)} = 1$.

For the determination of P_{i1}, and P_{i2}, we consider that $b_i^{(1)} = 1$, and that N_1 undesired users sent a 1.

A. Expression of P_{i1}

P_{i1} is the probability for an undesired user #j who send a "0" to create an interference of "-1" on user #1. Consequently, user #j must verify two conditions:

- he has one chip in common with user #1 code
- his datum is detected as a "1" instead of a "0".

So, we can write:

$$P_{i1} = P(\int_0^T C_1(t).C_j(t).dt = 1 \cap Z_i^{(j)} \ge S_u / (b_i^{(j)} = 0 \cap b_i^{(1)} = 1))$$
$$= p_1 \times P(Z_i^{(j)} \ge S_u / (\int_0^T C_1(t).C_j(t).dt = 1 \cap b_i^{(j)} = 0 \cap b_i^{(1)} = 1))$$

When $\int_0^T C_1(t).C_j(t).dt = 1$, there is an overlapping between user #1's and user #j's codes. Consequently, the user #1 (who send a datum "1") generates an interference of value +1 on user #j. Thus, the contribution I'_{CCRj} of the others users (i.e. the N_1 undesired users who send a "1") must be greater than $S_u - 1$. Thus:

$$P(Z_i^{(j)} \ge S_u / (\int_0^T C_1(t).C_j(t).dt = 1 \cap b_i^{(j)} = 0 \cap b_i^{(1)} = 1)$$
$$= P(I'_{CCRj} \ge S_u - 1)$$

An undesired user who send a 1, generates an interference of "+1" or of "+2" on user #j, with the probability p_1 and p_2 respectively. So the probability to have i_1 and i_2 users interfering with a value "+1" and "+2" respectively, among the N_1 undesired users who send a "1" is described by a trinomial rule, and can be expressed as:

$$\frac{N_1!}{i_1! i_2! (N_1 - i_1 - i_2)!} p_1^{i_1} p_2^{i_2} (1 - p_1 - p_2)^{N_1 - i_1 - i_2}$$

Moreover, we have:

$P(I'_{CCRj} \geq S_u - 1) = 1 - P(I'_{CCRj} < S_u - 1)$

and $I'_{CCRj} = i_1 + 2i_2$ when considering i_1 and i_2 users interfering with a value "+1" and "+2" respectively. So :

$$P(I'_{CCRj} \geq S_u - 1) = 1 - P(i_1 + 2i_2 < S_u - 1)$$
$$= 1 - \sum_{i_1=0}^{S_u-2} \sum_{i_2=0}^{\lfloor (S_u-2-i_1)/2 \rfloor} \left[\frac{N_1!}{i_1! i_2! (N_1 - i_1 - i_2)!} p_1^{i_1} p_2^{i_2} (1 - p_1 - p_2)^{N_1 - i_1 - i_2} \right]$$

Consequently, we finally get :

$$P_{i1} = p_1 \times \left(1 - \sum_{i_1=0}^{S_u-2} \sum_{i_2=0}^{\lfloor (S_u-2-i_1)/2 \rfloor} \left[\frac{N_1!}{i_1! i_2! (N_1 - i_1 - i_2)!} p_1^{i_1} p_2^{i_2} (1 - p_1 - p_2)^{N_1 - i_1 - i_2} \right] \right) \quad (4)$$

B. Expression of P_{i2}

P_{i2} is the probability for an undesired user #j who sends a "0" to create an interference of "-2" on user #1.

Consequently, user #j must verify two conditions:

- he has two chips in common with user #1's code
- his datum is detected as a "1" instead of a "0".

So, we can write:

$$P_{i2} = P\left(\int_0^T C_1(t).C_j(t).dt = 1 \cap Z_i^{(j)} \geq S_u / (b_i^{(j)} = 0 \cap b_i^{(1)} = 1)\right)$$
$$= p_2 \times P\left(Z_i^{(j)} \geq S_u / \left(\int_0^T C_1(t).C_j(t).dt = 1 \cap b_i^{(j)} = 0 \cap b_i^{(1)} = 1\right)\right)$$

When $\int_0^T C_1(t).C_j(t).dt = 2$, user #1 (who send a datum "1") generates an interference of value +2 on user #j. Thus, the contribution I'_{CCRj} of the other users (i.e. the N_1 undesired users who send a "1") must be greater than $S_u - 2$. Thus:

$$P\left(Z_i^{(j)} \geq S_u / \left(\int_0^T C_1(t).C_j(t).dt = 1 \cap b_i^{(j)} = 0 \cap b_i^{(1)} = 1\right)\right)$$
$$= P(I'_{CCRj} \geq S_u - 2)$$

Moreover, we have:

$$P(I'_{CCRj} \geq S_u - 2) = 1 - P(i_1 + 2i_2 < S_u - 2)$$
$$= 1 - \sum_{i_1=0}^{S_u-3} \sum_{i_2=0}^{\lfloor (S_u-3-i_1)/2 \rfloor} \left[\frac{N_1!}{i_1! i_2! (N_1 - i_1 - i_2)!} p_1^{i_1} p_2^{i_2} (1 - p_1 - p_2)^{N_1 - i_1 - i_2} \right]$$

Consequently, we finally get :

$$P_{i2} = p_2 \times \left(1 - \sum_{i_1=0}^{S_u-3} \sum_{i_2=0}^{\lfloor (S_u-3-i_1)/2 \rfloor} \left[\frac{N_1!}{i_1! i_2! (N_1 - i_1 - i_2)!} p_1^{i_1} p_2^{i_2} (1 - p_1 - p_2)^{N_1 - i_1 - i_2} \right] \right) \quad (5)$$

C. Expression of P_e

P_e is the error probability for user #1. P_e can be written:

$$P_e = \frac{1}{2} P(Z_i^{(1)} < S_d / b_i^{(1)} = 1)$$

We first consider that N_1 undesired users sent a "1". The probability to have exactly N_1 undesired users who sent a "1" is:

$$P(N_1) = \binom{N-1}{N_1} (1/2)^{N_1} \times (1/2)^{N-1-N_1} = \binom{N-1}{N_1} (1/2)^{N-1} \quad (6)$$

Thus

$$P_e = \frac{1}{2} \sum_{N_1=0}^{N-1} P(N_1) \times P(Z_i^{(1)} < S_d / N_1 \cap b_i^{(1)} = 1) \quad (7)$$

By considering N_{21} and N_{22} users creating interference of "-1" and "-2" on user #1 respectively, we can write:

$$P(Z_i^{(1)} < S_d / N_1 \cap b_i^{(1)} = 1) = P(W - N_{21} - 2N_{22} < S_d)$$
$$= P(N_{21} + 2N_{22} \geq W - S_d + 1)$$

Moreover, we have shown that an undesired user who sends a 0, generates an interference of "-1" and "-2" on user #1, with the probability P_{i1} and P_{i2} respectively. So the probability to have N_{21} and N_{22} users interfering with a value "-1" and "-2" respectively, among the $N - 1 - N_1$ undesired users who send a "0", can be expressed as:

$$\frac{(N-1-N_1)!}{N_{21}! N_{22}! (N - 1 - N_1 - N_{21} - N_{22})!} P_{i1}^{N_{21}} P_{i2}^{N_{22}} (1 - P_{i1} - P_{i2})^{N_1 - 1 - N_{21} - N_{22}}$$

Thus, we get :

$$P(Z_i^{(1)} < S_d / N_1 \cap b_i^{(1)} = 1) = 1 - P(N_{21} + 2N_{22} < W - S_d + 1)$$
$$= 1 - \sum_{N_{21}=0}^{W-S_d} \sum_{N_{22}=0}^{\lfloor (W-S_d-N_{21})/2 \rfloor}$$
$$\left[\frac{(N-1-N_1)!}{N_{21}! N_{22}! (N - 1 - N_1 - N_{21} - N_{22})!} P_{i1}^{N_{21}} P_{i2}^{N_{22}} (1 - P_{i1} - P_{i2})^{N_1 - 1 - N_{21} - N_{22}} \right] \quad (8)$$

We finally obtain the expression of the error probability from (6), (7) and (8) :

$$P_e = \left(\frac{1}{2}\right)^N \sum_{N_1=0}^{N-1} \binom{N-1}{N_1} \times \left(1 - \sum_{N_{21}=0}^{W-S_d} \sum_{N_{22}=0}^{\lfloor (W-S_d-N_{21})/2 \rfloor} \right.$$
$$\left. \left[\frac{(N-1-N_1)!}{N_{21}! N_{22}! (N - 1 - N_1 - N_{21} - N_{22})!} P_{i1}^{N_{21}} P_{i2}^{N_{22}} (1 - P_{i1} - P_{i2})^{N_1 - 1 - N_{21} - N_{22}} \right] \right) \quad (9)$$

IV. Results

Fig.3 shows the comparison between theoretical and simulated results for the PIC receiver for a PC(25,5) with *N=5* users, versus the threshold level values of the desired and undesired users' CCR respectively. We can first point out that the theoretical expression (9) correctly describes the PIC receiver performance.

In addition to that, we can observe that the BER decreases when S_u increases and S_d decreases. Thus, we can deduce that $S_u=P$ and $S_d=1$ are the optimal threshold levels. Indeed, for a CCR, the lowest error probability is obtained for $S_u=P$, so for such threshold level, the non-desired users are better estimated. Moreover, as the interference on the desired user is negative, the optimal threshold for the desired user is the smallest positive number, that is $S_d=1$.

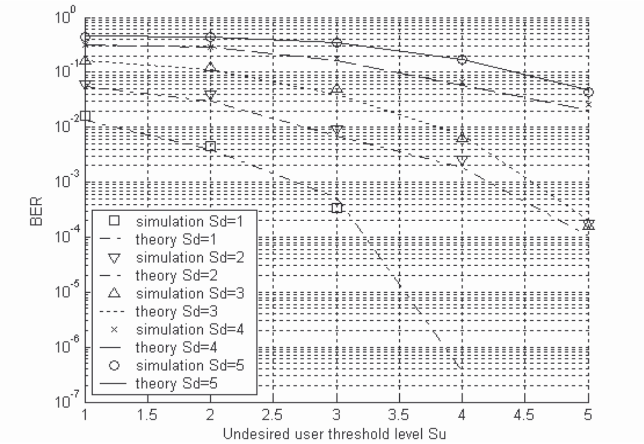

Figure 3. Simulated and theoretical PIC performances for PC (25,5) with N=5

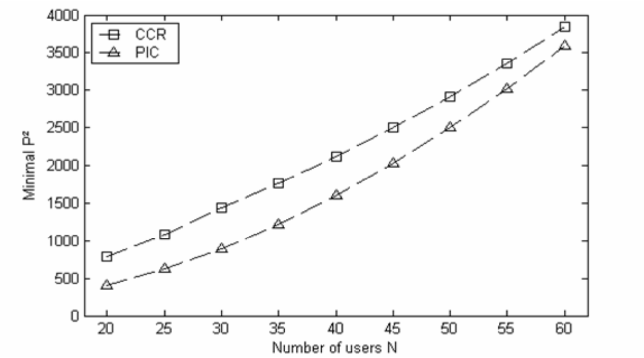

Figure 4. Minimal code length P^2 for BER<10^{-9} for PC with CCR and PIC

Furthermore, we can observe that for S_u=5 and S_d=1, the theoretical and simulated BER are null. It can be verified that, for all the values of P, the BER is null when considering the optimal threshold levels. Indeed, in order to have undesired users detected as "1" instead of "0",we must have for each of these users: $I_{CCRj} \geq S_u = P$, by considering the optimal threshold level. As the maximum interference contribution from one given user is *2*, there must be at least $\lceil P/2 \rceil$ users having sent a "1" to create errors on some undesired users, where $\lceil x \rceil$ denotes the smallest integer not less than x. Moreover, there is an error on the desired user if $I<S_d-P$, i.e. $|I| \geq P$ when considering the optimal threshold level S_d=1. Therefore there must be at least $\lceil P/2 \rceil$ undesired users interfering on the desired user. On the whole, to fit with the error conditions with the PIC receiver, there must be at least $\lceil P/2 \rceil$ users that sent a "*0*", and $\lceil P/2 \rceil$ users that sent a "*1*", so there must be at least $2 \times \lceil P/2 \rceil$. As P is a prime number, there must be at least $P+1$ different active users in the network. But, there are at most only P possible users in the code set. Thus, errors can never occur.

Thus, in spite of the high MAI due to high cross-correlation value, the PIC receiver applied to any PC leads to an error free transmission link in the noiseless case.

In addition to that, we can remark that for S_u=4 and S_d=1, the theoretical analysis predicts a non null BER whereas simulated BER seems to be null (we get no error for 10^{10} bits). Indeed, the theoretical probability is an upperbound. We did not take into account in our theoretical analysis that there is one user in each PC family set, who is always well detected (and thus can never be an interfering user for the desired user) and whose maximum interference on the undesired users is "1".

In order to evaluate the PIC benefit with PC, we have plotted on fig. 4 the minimal code length P^2 required to have a BER <10^{-9} for the optimal threshold levels, as a function of users number N. Indeed, due to electrical bandwidth limitation, the maximum data rate is inversely proportional to the code length. Thus, the code length reduction is a main ojective for practical OCDMA system. We can observe that the PIC receiver permits decreasing the code length required compared to CCR of about 40%. This decrease permits having more flexibility as regards to the electronic bandwidth.

V. Comparison with OOC

The first studied codes for OCDMA system are the Optical Orthogonal Codes (OOC) [4]. These unipolar code sequence are defined by $(F, W, \lambda_a, \lambda_c)$ where F is the sequence length, and W is the weight. λ_a and λ_c are the auto and crosscorrelation constraints. Contrary to PC whose number of users is given by the code weight, the maximum number of users N in the OOC class with $\lambda_a = \lambda_c = 1$ is defined as:

$$N \leq \left\lfloor \frac{(F-1)}{W(W-1)} \right\rfloor \tag{10}$$

We have previously shown [13] that the PIC receiver also permits reducing the required code length for OOC.

Consequently, in order to evaluate the most suitable code family, we have plotted on fig 5, the minimal code length required for the OOC and the PC to perform a given error probability with PIC receiver, as a function of the targeted number of users.

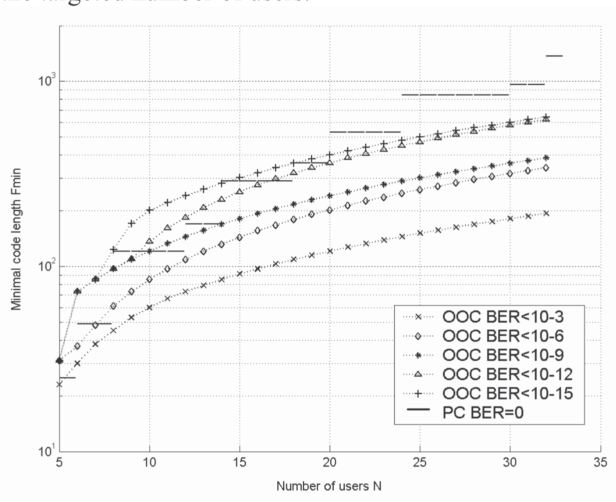

Figure 5. Minimal code lengths for OOC and PC with a PIC receiver

First, it must be noted that, as the use of PC along with PIC leads to an error free link, the minimal code performs whichever targeted error probability. On the contrary, as OOC leads to non-null error probability, the minimal code depends on the targeted BER: the smaller the aimed probability of error, the longer the minimum code length.

Besides, as the PC are constructed with a prime number, only some code lengths are available, that is why the PC curve is discontinuous.

Secondly, we can observe that, for a small number of users, the PC requires a smaller code length so is more efficient than the OOC. On the contrary, for a high number of users, OOC are more efficient. Finally, for a medium number of users, OOC are more interesting when the targeted error probability is high, but PC are more interesting for low error probability because of their null error probability. So, the noiseless study leads to the conclusion that, for a classical optical link (BER~10^{-9}), the OOC are more suitable than PC when the number of users is bigger than 13.

However, to assess this result, the noisy case must be evaluated. In this paper, we present results for a strong noise contribution, i.e. for a Signal to Noise Ratio SNR=5dB. We have considered the optimal OOC and PC required for an error probability smaller than 10^{-3} and 10^{-15} in the noiseless case. These codes values are reported in table I.

We have plotted on fig 6, for each number of users, the error probability obtained by the optimal OOC and PC for a given SNR=5dB. The OOC error probabilities have been obtained using the theoretical expression (24) in [14] for the optimal threshold level. The PC error probabilities have been obtained by simulation and correspond to the optimal threshold level.

TABLE I
THE MINIMAL CODES (F,W) FOR A TARGETED NUMBER OF USERS N

N	7	17	23	31
OOC 10^{-3}	(38,2)	(103,3)	(139,2)	(187,3)
OOC 10^{-15}	(85,4)	(341,5)	(461,5)	(621,5)
PC	(49,7)	(289,17)	(529,23)	(961,31)

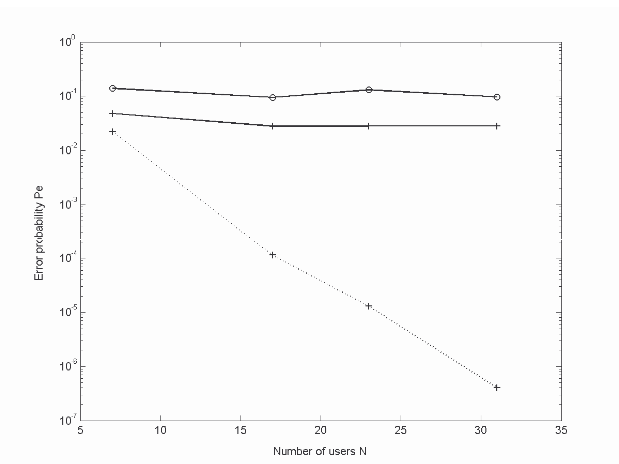

Figure 6. Error probability for the optimal OOC and PC with a PIC receiver, for SNR=5dB

TABLE II
THE MINIMAL REQUIRED SNR FOR $P_E<4.10^{-7}$

	OOC(373,3)	OOC(600,4)	OOC(621,5)	PC(961,31)
SNR(dB)	24	19	16	5

First, we can note that the curve is constant as a function of *N* for the OOC corresponding to a targeted BER. Indeed, the results are obtained for the optimal code leading to the same error probability in the noiseless case, so the amount of MAI is equivalent. Moreover, the code weight of the different OOC is quite the same for a targeted BER, thus, the noise contribution has the same impact on the performance.

Furthermore, we can remark that the codes for 10^{-15} lead to better performance than the one for 10^{-3} in the noiseless case. This is due to the fact that the weight is bigger for the first one, and that the code weight increasing improves the performance for a given SNR [14].

Besides, it can be seen that the PC performance improves when the number of users increases. Indeed, the PC code weight increases with the number of users. This makes the decision less sensitive to the noise perturbation. (It can be noted that, due to their construction, the OOC would require a much higher code length to obtain the same code weight than the PC, so it is not worth to study this case). In addition, as the PC weight is higher than the OOC one for whichever number of users, the PC obtains better performance than the OOC.

To sum up, for a given number of users, and a given SNR, the PC leads to better performance, but at the cost of an increased length. Consequently, to estimate the tradeoff between the required SNR and code length, we have searched the minimal SNR required for the OOC to obtain the same performance than the PC for *N*=31 users, and a SNR=5 dB, i.e. 4.10^{-7} according fig 6.

We can observe on table II that, compared to the OOC, the PC needs a higher code length (1.5 to 3 as much higher), but a lower SNR (11 to 19 dB reduction). This highlights the interest of the PC, which permits significantly reducing the required SNR for a targeted performance, at the cost of a reasonable code increase. This is an important advantage for practical OCDMA systems regards to implementation cost.

VI. CONCLUSION

We have evaluated the Parallel Interference Cancellation receiver (PIC) efficiency in a DS-OCDMA link with Prime Codes (PC). The analytic expression of the error probability upperbound in the case of Prime Codes has been established. From numerical calculation and simulation, we have proved the reliability of the theoretical analysis. Moreover, we have shown theoretically that, in spite of the high cross-correlation value of the PC, the PIC receiver permits removing the Multiple Access Interference (MAI) and permits obtaining an error free transmission link, in the noiseless case, for whichever Prime Code.

From this theoretical analysis, we have shown that by reducing the code length, the PIC receiver brings flexibility regards to the electronic bandwidth. Besides, we have compared the PC and OOC efficiency, and pointed out that, for a targeted performance, the PC require a slightly higher code length but a significantly reduced SNR. This highlights the benefits of using PC for application in OCDMA systems, and this is an encouraging result for further development of these codes, in particular for generating 2D codes which are currently intensively studied and allow to further relax the temporal code length.

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Claire Goursaud-Brugeaud was born in Limoges, France on October 11,1980. She received her Diplôme d'Ingénieur degree from the Ecole Nationale Supérieure d'Ingénieurs de Limoges (ENSIL), University of Limoges, France in 2003. She received the PhD degree on optical and microwaves communications in 2006.

She is currently an assistant professor in Limoges University. She teaches mathematics, electronics, and optics. Her research work deals with the study of the digital signal processing techniques applied to Optical Code Division Multiple Access.

Anne Julien-Vergonjanne was born in Nîmes, France on December 29,1961. She received the PhD degree on optical and microwaves communications in 1997.The major fields she has studied concerns microwave filtering and microwave material characterization.

She is currently the leader of Electronics and Telecommunications department at ENSIL, Limoges University. She teaches digital communication and signal processing. Her research activities include channel coding, multiuser detection and multimedia transmission for optical communications.

Pr Julien-Vergonjanne is a member of IEEE communication society.

Jean Pierre Cances was born in Brive, France, on April 17, 1966. He received the Eng. degree from the Ecole Nationale Supérieure des Télécommunications de Bretagne (ENSTBr) in 1990 and the Ph.D. degree from the Ecole Nationale Supérieure des Télécommunications (ENST-Télécom Paris) in 1993. He also received the teaching degree Agregation in physics in 1993 and the teaching degree CAPES in mathematics in 1994.

He is currently a full professor at ENSIL Limoges University where he teaches signal processing. His research activities include channel coding, iterative processing for MIMO communications and optical communications.