Precoded Spatial Multiplexing MIMO Systems in Time-Varying Fading Channel

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***Abstract*—**Conventional precoded spatial multiplexing multiple-input multiple-output (MIMO) systems using limited feedback are mainly based on the notion of time invariant channels throughout transmission. Consequ-encely, the precoding matrix can be found during the training symbols and used over the subsequent data symbols. In this paper, a more practical system where the channel varies from one block of symbols to another is considered. In such scenario, the precoding matrix designed at the receiver based on the previous training symbols becomes outdated, which results in significant system performance degradation. In order to avoid this problem, and reduce performance degradation, we propose the use of a Kalman filter linear predictor at the receiver to provide the transmitter with the precoding matrix for the next block of symbols. The performance of this method is assessed using computer simulation, and the obtained results for the proposed channel prediction demonstrate improved bit error rate performance for time-varying Rayleigh fading channels.

***Index Terms*—**feedback delay, Kalman filter, limited feedback, MIMO systems, spatial multiplexing

I. Introduction

Spatial multiplexing, in which a bit stream is demultiplexed into multiple substreams that are sent over different antennas, allows MIMO wireless systems to obtain high spectral efficiency. However, since the transmitted signals and the channel are not well matched, degradation in system performance is unavoidable. Linear precoding is a technique employed to allow the transmitter to adapt to the propagation conditions, combat rank deficiency problems and reduce the probability of error [1], [2]. However precoding requires some form of knowledge of the channel conditions, which is often called channel state information (CSI) at the transmitter (CSIT). Reciprocity and feedback are the main techniques used to obtain CSIT. Reciprocity involves using the uplink channel information to estimate the downlink channel, and it is used in time division duplex (TDD) systems, where the downlink and the uplink channels are almost identical. In contrast, feedback requires sending the downlink channel information back to the transmitter through the feedback channel. Feedback technique is used with systems using frequency division duplex (FDD), as in FDD systems the downlink and uplink channels are in general highly uncorrelated since they are separated in frequency. However, in the feedback technique, there will always be a feedback delay between the time when the channel information is obtained and when it is available at the transmitter. The feedback information accuracy will depend on this delay and on the channel estimation technique. Channel estimation at the receiver is the starting point for obtaining CSIT and its accuracy depends strongly on the estimation technique. Furthermore, for a time varying channel this information must be continuously updated, otherwise the outdated channel information will significantly affect the CSIT accuracy, and will result in system performance degradation.

The design of an efficient feedback scheme that provides reliable CSI to the transmitter necessitates firstly minimising the amount of information to be fed back to the transmitter through the feedback channel, and secondly solving the feedback delay problem. The first issue has been extensively studied in past research work such as [3]-[6], where the precoding matrix is chosen from a common codebook, known in advance at the transmitter and the receiver, and only the index of the precoding matrix is fed back to the transmitter through a limited rate feedback channel. As with the second issue of feedback delay in the feedback channel, a prediction scheme for time varying MIMO channel has been proposed in [7]. The proposed scheme is an extension of the geodesic interpolation method, which is used to predict the future precoder directly without going through the prediction of the channel matrix. The channel estimation errors and the quantization error are not taken into account when evaluating the performance of this scheme. A method based on Markov chain theory for analysing the effect of feedback delay on a transmit beamforming system with limited feedback has been proposed in [8]. The results presented show that the capacity gain with respect to the case of no feedback diminishes at least exponentially with the feedback delay. However, the channel has been assumed to be perfectly known at the receiver. The problem of finite rate feedback for spatially correlated Rayleigh fading Multiple Input Single Output (MISO) channel with estimation errors at the receiver and feedback delay was addressed in [9], and a codebook design algorithm that minimises the loss in ergodic capacity was proposed. In

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[10], it was shown that the performance degradation of MIMO systems in the presence of feedback delay is reduced by predicting the channel at future times when it will be used, and feeding it back from the receiver to the transmitter. In [11], it was also reported that the performance degradation of MIMO systems in the presence of feedback delay is reduced by using a receiver based on zero-forcing (ZF) or minimum mean square error (MMSE) criteria instead of those for Singular Value Decomposition (SVD) criterion. A method that combines channel prediction with ZF or MMSE-based receiver weights was proposed in [12], where the channel prediction is performed using the Wiener filter. However, the Wiener filter is strongly constrained on assuming stationary and infinite time observation. Furthermore, infinite rate feedback channel has been assumed.

This paper proposes a prediction technique capable of improving the BER performance of precoded MIMO systems even in the system with high mobility. The main contribution of this paper includes the application of Kalman filter linear prediction at the receiver to predict the channel state require to design the precoding matrix of the next block, whose index is fed back to the transmitter, thus mitigating the feedback delay problem. We used a receiver based on ZF and MMSE criteria. Simulation results demonstrate that the proposed method improves the bit error rate (BER) performance significantly in time varying Rayleigh fading channels.

The rest of this paper is organized as follows: the system model is presented in Section II. Simulation results are presented in Section III and concluding remarks are given in Section IV.

II. System Model

A precoded spatial multiplexing system with limited feedback is shown in Fig. 1. We consider a system with N_t transmit and N_r receive antennas. The input bit stream is modulated and then demultiplexed into M substreams, where the number of substreams $M < N_t$ and $M \le N_r$. Let the vector $\boldsymbol{s}[n]=\left[s_1(n),s_2(n),\ldots,s_M(n)\right]^T$ denotes the M x1 transmitted symbol vector, where T denotes transpose operation, and n is the time index. We assume that $E\left[\boldsymbol{ss}^H\right]=\left(\varepsilon_s / M\right)\mathbf{I}_M$ in order to constrain the transmitted power, where $(.)^H$ refers to matrix conjugate transposition, $\mathbf{I}_M$ is the $M \times M$ identity matrix, ε_s denotes the transmit energy, and $E[.]$ represents the statistical expectation operator. The symbol vector $\boldsymbol{s}[n]$ is multiplied by the $N_t \times M$ precoder matrix $\mathbf{F}[n]$ generating a length N_t vector $\boldsymbol{x}[n]=\mathbf{F}[n]\boldsymbol{s}[n]$, where $\mathbf{F}[n]\in u\left(N_t, M\right)$, the set of $N_t \times M$ complex unitary matrices. $\mathbf{F}[n]$ is selected at the receiver from a finite set of possible precoding matrices $\mathcal{F}=\left\{\mathbf{F}_1,\mathbf{F}_2,\ldots,\mathbf{F}_N\right\}$ represented by a limited number of bits $B\left(B=\log_2 N\right)$ and sent to the transmitter through a limited feedback channel. In the published works on limited feedback for spatial multiplexing MIMO systems [3]-[6], the precoder matrix is chosen at the receiver from a finite length codebook $\mathcal{F}$ using the current channel state $\mathbf{H}[n]$ and perfect channel knowledge at the receiver is assumed.

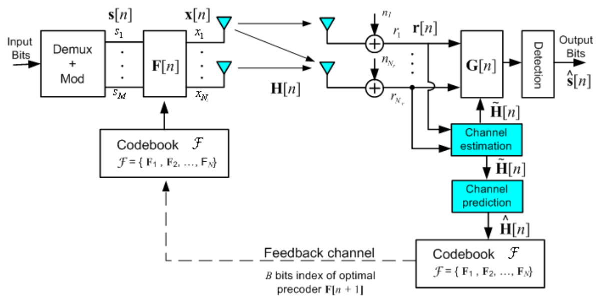

Fig. 1. Precoded MIMO system with limited feedback.

In this work, however, we consider a more practical time varying channel. A Kalman filter is used to estimate the channel at the receiver to predict the future state of the channel which is used to design the precoder matrix. It is known that a dynamic system can be modelled as an autoregressive (AR) process. A Qth-order AR model for $\boldsymbol{h}[n]$ is presented as:

$$\boldsymbol{h}[n+1]=\sum_{q=1}^{Q}\mathbf{A}(q)\boldsymbol{h}[n-q+1]+\boldsymbol{w}[n] \tag{1}$$

where $\mathbf{A}(q)$ are the AR process coefficients, and $\boldsymbol{w}[n]$ is vector noise process. The parameters of the AR process can be obtained by solving the Yule-Walker equation. The choice of Q is a trade-off between the accuracy of the model (1) and the complexity in estimating its parameters. In this work, for simplicity, we have modelled the channel as a first order AR process. The state space equations describing the channel are expressed as:

$$\boldsymbol{h}[n+1]=\mathbf{A}\boldsymbol{h}[n]+w[n] \tag{2}$$

$$\boldsymbol{y}[n]=\boldsymbol{C}[n]\boldsymbol{h}[n]+v[n] \tag{3}$$

where $\boldsymbol{h}[n]$ represents the $N_t N_r \times 1$ channel taps vector, $\mathbf{A}$ is a known $N_t N_r \times N_t N_r$ matrix that denotes the time varying transition matrix, and $\boldsymbol{C}[n]$ is a known $N_r \times N_t N_r$ measurement matrix. The $N_r \times 1$ vector $\boldsymbol{v}[n]$ is the measurement noise and the

$N_t N_r \times 1$ vector $\boldsymbol{w}[n]$ is called the process noise. The noise vectors $\boldsymbol{w}[n]$ and $\boldsymbol{v}[n]$ are mutually uncorrelated white noise sequences with covariance matrices $\Phi_w[n]$ and $\Phi_v[n]$ so we can write $E\left[\boldsymbol{v}[n]\boldsymbol{w}^H[m]\right]=0$, for all n and m. A first order AR model provides an adequate model for time varying channels [13]. Consequently, $\mathbf{A}$ is a diagonal matrix of autoregressive model factor $\alpha = E\left[h_{ij}[n+1]*h_{ij}^*[n]\right]$. According to Jakes' model:

$$\alpha = E\left[h_{ij}[n+1]*h_{ij}^*[n]\right] = J_0\left(2\pi f_d T_s\right) \quad (4)$$

where $J_0(.)$ denotes the zeroth-order Bessel function of the first kind, f_d is the Doppler frequency, and T_s is the symbol duration. For a spatial multiplexing MIMO system with N_t transmit and N_r receive antennas, the measurement matrix $\boldsymbol{C}[n]$ is given as:

$$\boldsymbol{C}[n] = \boldsymbol{x}^T[n] \otimes \mathbf{I}_{N_r} \quad (5)$$

$$\boldsymbol{x}[n] = \left[x_1[n], x_2[n], \ldots, x_{N_t}[n]\right]^T \quad (6)$$

where $x_j[n]$ is the transmitted symbol from antenna j ($j = 1, 2, \ldots, N_t$) at time n, and $\otimes$ denotes the Kronecker product.

In this work we consider a burst-mode communication system where the transmitted data is divided into frames, each of which contains multiple symbols. We also consider a precoded MIMO system where the channel remains unchanged for the duration of the frame; however, it varies from frame to frame. The codebook's design and the precoding selection criteria to select the codeword from the codebook are discussed in the following subsections.

A. Precoder Selection Criteria

A plethora of precoder (codeword) selection criteria for spatial multiplexing systems using linear receivers based on perfect CSI at the receiver have been proposed in [5] and [6] to select the optimal precoding matrix from a given codebook by searching through all codebook matrices. The design of precoders using linear receivers can be achieved in different ways depending on the used decoding matrix $\mathbf{G}$, and the precoding matrices are constrained to be unitary (i.e. $\mathbf{F}_i^H \mathbf{F}_i = \mathbf{I}_M, i = 1, 2, \ldots, N$). The proposed selection criteria in [5] and [6] can be briefly summarized as follows:

- For linear ZF receivers, the precoding matrix is chosen to maximize the minimum singular value of $\mathbf{HF}$. This is known as minimum singular value selection criteria (MSV-SC).

For the MSV-SC: Select $\mathbf{F}$ such that

$$\mathbf{F} = \arg\max_{\mathbf{F}_i \in \mathcal{F}} \lambda_{\min}\left(\mathbf{HF}_i\right) \quad (7)$$

where $\lambda_{\min}$ is the minimum singular value of the effective channel matrix $\mathbf{HF}_i$.

- For linear MMSE receivers, the precoding matrix is chosen to minimize either the trace of the mean square error matrix (MMSE-trace Selection) or the determinant of the mean square error matrix (MMSE-det. Selection)

For mean square error selection criterion (MSE-SC): $\mathbf{F}$ is chosen according to:

$$\mathbf{F} = \arg\min_{\mathbf{F}_i \in \mathcal{F}} \mathcal{M}\left(MSE\left(\mathbf{F}_i\right)\right) \quad (8)$$

where the mean squared error (MSE) for linear MMSE receiver is expressed as

$$MSE\left(\mathbf{F}\right) = \frac{\mathcal{E}_s}{M}\left(\mathbf{I}_M + \frac{\mathcal{E}_s}{MN_0}\mathbf{F}^H\mathbf{H}^H\mathbf{HF}\right)^{-1} \quad (9)$$

where $\mathbf{H}$ is the channel matrix, and $\mathcal{M}(.)$ is either trace (*tr*) or determinant (*det*).

Instead of the selection criteria based on the indirect performance indicator in [5], a precoder selection criterion that directly uses the bit error rate (BER) of the system has been proposed in [6]. For the BER-based selection criterion the precoder is chosen according to [6]

$$\mathbf{F} = \arg\min_{\mathbf{F}_i \in \mathcal{F}} \mathrm{BER}\left(\mathbf{H}, \mathbf{F}\right) \quad (10)$$

where, $\mathrm{BER}\left(\mathbf{H},\mathbf{F}\right)$ denotes the BER averaged over M data streams when the channel realisation is $\mathbf{H}$ and the precoder matrix is $\mathbf{F}$.

- For the linear ZF receivers, the average BER over M data stream is given by

$$\mathrm{BER}^{ZF}\left(\mathbf{H},\mathbf{F}\right) = \frac{1}{M}\sum_{k=1}^{M}\phi\left(\gamma_k^{ZF}\right) \quad (11)$$

where $\phi(\gamma)$ is a finite sum of Gaussian-Q functions, and the signal-to-noise ratio (SNR) for the kth data stream is

$$\gamma_k^{ZF} = \frac{E_s}{N_0\left[\mathbf{F}^H\mathbf{H}^H\mathbf{HF}\right]_{k,k}^{-1}} \quad (12)$$

- For the linear MMSE receiver, the average BER is given as

$$\mathrm{BER}^{MMSE}\left(\mathbf{H},\mathbf{F}\right) = \frac{1}{M}\sum_{k=1}^{M}\phi\left(\gamma_k^{MMSE}\right) \quad (13)$$

and the SNR is given by

$$\gamma_k^{MMSE} = \frac{E_s}{N_0 \left[\mathbf{F}^H \mathbf{H}^H \mathbf{HF} + \left(N_0 / E_s \right) \mathbf{I}_M \right]_{k,k}^{-1}} - 1 \quad (14)$$

B. Codebook Design Criteria

For infinite-rate feedback ($B = \infty$), the optimal precoder for all selection criteria is given in [5] for comparison purpose. Let the eigenvalue decomposition of $\mathbf{H}^H \mathbf{H}$ be given by

$$\mathbf{H}^H \mathbf{H} = \mathbf{V}_H \; \Sigma \; \mathbf{V}_H^H \quad (15)$$

where $\mathbf{V}_H \in \mathcal{U}\left(N_t, N_r\right)$, and Σ is an $N_t \times N_t$ diagonal matrix that contains on its diagonal the eigenvalues of $\mathbf{H}^H \mathbf{H}$ arranged in decreasing order: $\lambda_1 \geq \lambda_2 \ldots \geq \lambda_{N_t}$. Then the optimal precoder is given as [5]

$$\mathbf{F}_{opt} = \bar{\mathbf{V}}_H \quad (16)$$

where $\bar{\mathbf{V}}_H$ is a matrix constructed from the first M columns of $\mathbf{V}_H$.

For finite rate feedback, the codebook design criterion for each selection criterion is also given in [5], which can be summarised as follows:

- If MSV-SC or MMSE-SC (with a trace-based cost function) is used, the codebook $\mathcal{F}$ should be designed such that $\min_{\mathbf{F}_i \neq \mathbf{F}_j} d_{p2}\left(\mathbf{F}_i, \mathbf{F}_j\right)$ is maximised, where $d_{p2}\left(\mathbf{F}_i, \mathbf{F}_j\right)$ is the projection two-norm subspace distance defined as [14]

$$d_{p2}\left(\mathbf{F}_i, \mathbf{F}_j\right) = \left\| \mathbf{F}_i \mathbf{F}_i^H - \mathbf{F}_j \mathbf{F}_j^H \right\|_2 \quad (17)$$

- If MMSE-SC (with a determinant cost function) is used, the codebook $\mathcal{F}$ should be designed such that $\min_{\mathbf{F}_i \neq \mathbf{F}_j} d_{FS}\left(\mathbf{F}_i, \mathbf{F}_j\right)$ is maximised, where $d_{FS}\left(\mathbf{F}_i, \mathbf{F}_j\right)$ is the Fubini-Study distance defined as [14]

$$d_{FS}\left(\mathbf{F}_i, \mathbf{F}_j\right) = \arccos\left| \det\left(\mathbf{F}_i^H \mathbf{F}_j \right) \right| \quad (18)$$

A precoder codebook construction method based on the generalised Lloyds algorithm has been proposed in [6]. These codebooks were designed to maximize $\min_{\mathbf{F}_i \neq \mathbf{F}_j} d_c\left(\mathbf{F}_i, \mathbf{F}_j\right)$, where $d_c\left(\mathbf{F}_i, \mathbf{F}_j\right)$ is the chordal subspace distance defined as [14]

$$d_c\left(\mathbf{F}_i, \mathbf{F}_j\right) = \frac{1}{\sqrt{2}} \left\| \mathbf{F}_i \mathbf{F}_i^H - \mathbf{F}_j \mathbf{F}_j^H \right\|_F \quad (19)$$

In order to simplify the analytical solution inside the iterations of Lloyd's algorithm, (19) can be rewritten in the following convenient form

$$d_c\left(\mathbf{F}_i, \mathbf{F}_j\right) = \sqrt{tr\left(\mathbf{I}_M - \mathbf{F}_j^H \mathbf{F}_i \mathbf{F}_i^H \mathbf{F}_j \right)} \quad (20)$$

A comparison between these codebooks and the codebooks proposed in [5] is given in [6]. BER performance comparison shows that these codebooks perform approximately the same. Sample codebooks can be found in [15] and [16].

C. Channel Estimation and Prediction

The Kalman filter has been used to track the time varying channel in MIMO systems [13]; however, this has been the case only for unprecoded MIMO systems. In this paper, in addition to using the Kalman filter to estimate the flat fading channel in precoded MIMO systems, it is also used to predict the channel state for the next block, based on the collection of the past estimated channel values. The predicted channel state is required to design the precoding matrix for the next block, and the index for the precoding matrix is fed back to the transmitter through the limited feedback channel. Channel prediction is proposed in this paper to overcome the delay effect in the feedback channel in a precoded MIMO system. In order to utilise the Kalman filter (2), (3) are needed for the state and observation equations respectively. Furthermore, the assumption that the process noise and measurement noise variances in the state space model (2), (3) are known is commonly used. In this work the Kalman filter is employed as a training scheme to give the channel estimation and prediction. The frame format at the transmitter is depicted in Fig. 2. The first frame of length L_f symbols is used as a training frame. In the following frames L_p tracking symbols are periodically inserted per frame of length L_f . During the training period the transmitted symbols are known to the receiver. Then through the Kalman filter described in [17], the estimated channel can be obtained by the following recursive computation:

where the prediction part is given as

$$\tilde{\boldsymbol{h}}\left[n+1/n\right] = \mathbf{A}\boldsymbol{h}\left[n/n\right] \quad (21)$$

$$\mathbf{P}\left[n+1/n\right] = \mathbf{AP}\left[n/n\right]\mathbf{A} + \mathbf{\Phi}_w \quad (22)$$

$$\boldsymbol{\alpha}\left[n\right] = \boldsymbol{y}\left[n\right] - \boldsymbol{C}\left[n\right]\tilde{\boldsymbol{h}}\left[n+1/n\right] \quad (23)$$

$$\boldsymbol{K}[n] = \mathbf{P}\left[n+1\right]\boldsymbol{C}^H\left[n\right]\left[\boldsymbol{C}\left[n\right]\mathbf{P}\left[n+1/n\right]\boldsymbol{C}^H\left[n\right] + \mathbf{\Phi}_v\right]^{-1} \quad (24)$$

And the update part is given as

$$\tilde{\boldsymbol{h}}\left[n+1/n+1\right] = \tilde{\boldsymbol{h}}\left[n+1/n\right] + \boldsymbol{K}\left[n\right]\boldsymbol{\alpha}\left[n\right] \quad (25)$$

$$\mathbf{P}\left[n+1/n+1\right] = \left[\mathbf{I}\text{-}\boldsymbol{K}\left[n\right]\boldsymbol{C}\left[n\right]\right]\mathbf{P}\left[n+1/n\right] \quad (26)$$

where $\boldsymbol{K}\left[n\right]$ is the Kalman gain, $\mathbf{P}\left[n\right]$ is the correlation matrix of the error, and $\boldsymbol{\alpha}\left[n\right]$ is the innovations vector.

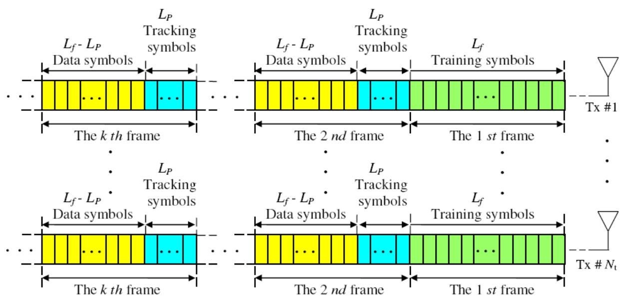

Fig. 2. Frame format at the transmitter.

Similar to [3]-[6], we assume that the receiver is capable of feeding back a finite number of bits to the transmitter through a zero error feedback channel. Moreover, we consider the feedback delay due to signal processing delay at both the receiver and transmitter, and the transmission delay. To overcome the effect of the feedback delay on the system performance, the Kalman filter is used to predict the channel information of the next frame $\mathbf{H}_p$ and the precoder matrix $\mathbf{F}[n]$ is selected at the receiver from the codebook $\mathcal{F}$ based on the predicted channel state information $\mathbf{H}_p$ using one of the precoder selection criteria stated before. Once the precoder is selected from the codebook, the index of this precoder is fed back to the transmitter through a finite rate feedback channel.

It should be noted that the Kalman filter is not used to predict the next block precoding matrix directly; however, it is used to predict the future channel state which is then used to choose the precoding matrix from the codebook. The reasons behind this are twofold; firstly, the codebook is designed offline according to some performance criteria (as described in subsection B) and the optimum precoding matrix is chosen from the codebook using one of the selection criteria stated in subsection A. Secondly, each precoding matrix in the codebook must be unitary (i.e. $\mathbf{F}_i^H\mathbf{F}_i = \mathbf{I}_M, i = 1,2,\ldots,N$). It is clear that these two conditions can not be guaranteed if the Kalman filter prediction is used to predict the precoding matrix directly.

Because our focus in this work is on mitigating the feedback channel delay in time varying MIMO channels, in this paper, we limit our discussion to MSV-SC and MSE-SC using the trace-based cost function proposed in [5], however, the proposed scheme can be used with all other selection criteria. Making use of the predicted channel $\mathbf{H}_p$, the linear precoder matrix $\mathbf{F}[n]$ can then be designed. When the predicted channel state $\mathbf{H}_p$ is used instead of the actual channel $\mathbf{H}$ then (7)-(9) becomes:

$$\mathbf{F} = \arg\max_{\mathbf{F}_i \in \mathcal{F}} \lambda_{\min}\left(\mathbf{H}_p\mathbf{F}_i\right) \tag{27}$$

where $\lambda_{\min}$ is the minimum singular value of the effective channel matrix $\mathbf{H}_p\mathbf{F}_i$.

For MSE-Sc $\mathbf{F}$ is chosen according to

$$\mathbf{F} = \arg\min_{\mathbf{F}_i \in \mathcal{F}} \mathcal{M}\left(MSE\left(\mathbf{F}_i\right)\right) \tag{28}$$

and the MSE for linear MMSE receiver is expressed as

$$MSE\left(\mathbf{F}\right) = \frac{\mathcal{E}_s}{M}\left(\mathbf{I}_M + \frac{\mathcal{E}_s}{MN_0}\mathbf{F}^H\mathbf{H}_p^H\mathbf{H}_p\mathbf{F}\right)^{-1} \tag{29}$$

The predicted precoder index is sent back to the transmitter. The received signal vector is assumed to be added with a noise vector $\boldsymbol{n}[n]$ whose entries are independent and distributed according to $\mathcal{CN}(0,N_0)$. Then the signal seen at the receiver can be written as:

$$\boldsymbol{r}[n] = \boldsymbol{C}[n]\boldsymbol{h}[n] + \boldsymbol{n}[n] \tag{30}$$

where $\boldsymbol{h}[n]$ is the $N_t N_r \times 1$ channel taps vector and $\boldsymbol{C}[n]$ is given by (5) and (6).

The estimated channel taps vector $\tilde{\boldsymbol{h}}[n]$ is changed to a matrix $\tilde{\mathbf{H}}$ of dimension $N_r \times N_t$ .Using the estimated channel matrix $\tilde{\mathbf{H}}$, the linear decoder applies an $M \times N_t$ matrix $\mathbf{G}[n]$ to $\boldsymbol{r}[n]$ to produce the vector $\hat{s}[n] = Q\left(\mathbf{G}[n]\boldsymbol{r}[n]\right)$, where $Q(.)$ is a function that performs a single dimensional maximum likelihood decoding for each entry of the vector. For a ZF linear decoder, $\mathbf{G}[n]$ is given as:

$$\mathbf{G}[n] = \left(\tilde{\mathbf{H}}\mathbf{F}\right)^+ = \left[\mathbf{F}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{F}\right]^{-1}\mathbf{F}^H\tilde{\mathbf{H}}^H \tag{31}$$

In contrast, for the MMSE linear decoder $\mathbf{G}[n]$ is given as:

$$\mathbf{G}[n] = \left[\mathbf{F}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{F} + \frac{MN_0}{\mathcal{E}_s}\mathbf{I}_M\right]^{-1}\mathbf{F}^H\tilde{\mathbf{H}}^H \tag{32}$$

where $(.)^+$ is the matrix pseudo-inverse, and $(.)^{-1}$ denotes the matrix inverse.

III. Simulation Results

Simulations were carried out to evaluate the system performance for the following scenarios; firstly, the CSI is assumed to be perfectly known and there is zero delay in the feedback channel, which represents the ideal channel case; secondly, the channel is estimated at the receiver using a Kalman filter and then the precoder matrix $\mathbf{F}[n]$ is designed at the receiver as a function of the estimated channel $\mathbf{H}[n]$, which is then fed back to the transmitter; finally, the channel is estimated at the receiver using Kalman filter and the predicted future channel state is fed back to the transmitter by selecting the precoder matrix from the codebook using the predicted channel state $\mathbf{H}_p[n]$ and (27)-(29). The codebooks used in the simulation are listed in [16]. A frame length L_f of 200 symbols is used, and L_p is taken as 16 symbols. 16 Quadrature Amplitude Modulation (QAM) was used as a modulation scheme unless otherwise specified to simulate M substreams precoding for an $N_t \times N_r$ MIMO wireless system. We considered a system with 2 GHz carrier frequency and a normalized Doppler frequency $f_d T_s = 10^{-2}$.

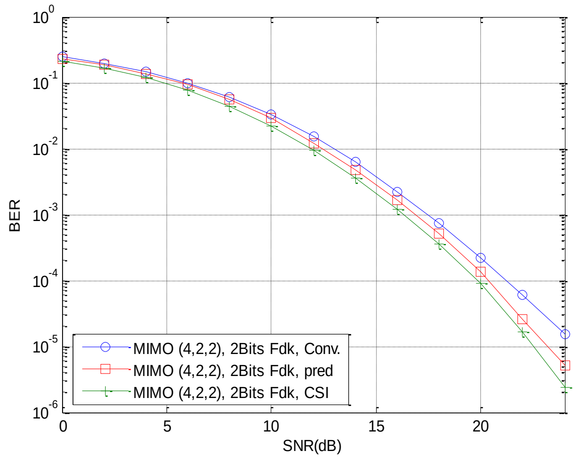

Fig. 3. BER comparison of conventional, CSI and prediction situations for a system with $(N_t, N_r, M) = (4,2,2)$, using 16QAM

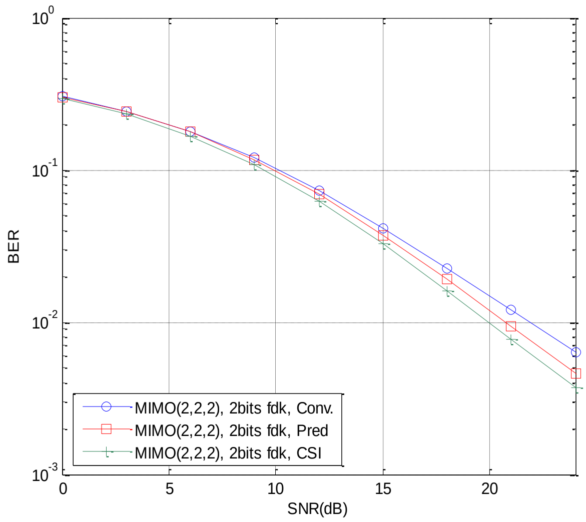

Fig. 4. BER comparison of conventional, CSI and prediction situations for a system with $(N_t, N_r, M) = (2,2,2)$, using 16QAM

Case 1: BER performance was obtained for perfect channel knowledge and delay-free feedback channel denoted by 'CSI'; the conventional case when there is a feedback delay denoted by 'Conv.', and for the Kalman filter based channel prediction denoted by 'Pred.'. The simulation results in Fig. 3 and Fig. 4 show the BER versus the SNR for 4×2 and 2×2 systems respectively using two substreams and two bits of feedback. ZF receiver employing a minimum singular value selection criterion (MSV-SC) was used for this scenario. It can be seen that channel prediction improves the system performance for both systems. It is also observed that at BER of 10^{-3}, the channel prediction scheme achieves ~ 1dB improvement over the conventional case for a 4×2 system. The performance improvement by the prediction scheme is due to mitigating the effect of delay in the feedback channel. However, it is still inferior to the unrealistic case of perfect CSI, which serves as the benchmark performance.

Case 2: The same scenario as for case 1 is simulated; however, in this case we investigated the impact of increasing the number of feedback bits on the BER performance using channel prediction for a 4×2 system. The simulation results are presented in Fig. 5. It can be seen that by increasing the number of feedback bits, (from 2 to 6 bits), $\sim$ 3dB improvement was achieved in BER performance. Additionally, it can be noted that using 6 bits feedback performs approximately the same as the optimal (infinite number of feedback bits) prediction 'Optimal, Pred.' precoding case for high SNR. This demonstrates that the system performance significantly improves as the number of feedback bits increases, and a satisfactory BER performance can be achieved with a reasonable number of bits (6 bits).

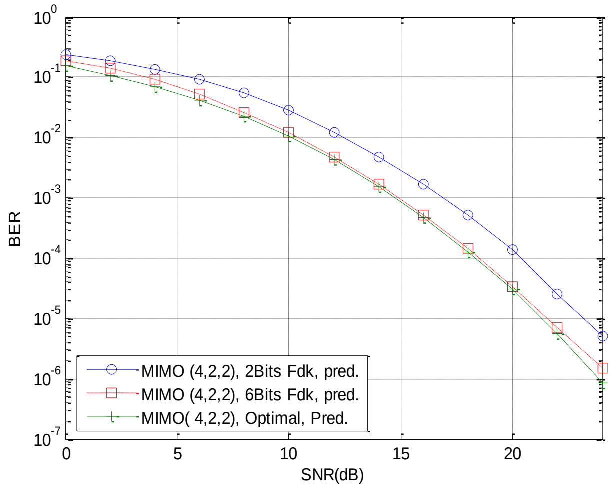

Fig. 5. Performance improvement with number of feedback bits, for a system with $(N_t, N_r, M) = (4,2,2)$, using 16-QAM

Case 3: In this case three precoding substreams on a 6x3 spatial multiplexing MIMO system was simulated using 4-QAM and 4 bits of feedback. The BER performance of linear ZF and linear MMSE receivers were compared. The MSV-SC was used with the ZF receiver, whereas the MSE-SC with trace-based cost function was used with the MMSE receiver. The results in Fig. 6 and Fig. 7 show the BER performance of perfect channel state information (CSI), conventional (Conv.), and prediction (Pred.) situations for both ZF and MMSE receivers, respectively. It is observed that the proposed prediction scheme outperforms the conventional case for both receivers. Furthermore, it can be observed that MMSE receiver performs better than ZF receiver at the expense of SNR knowledge at the receiver.

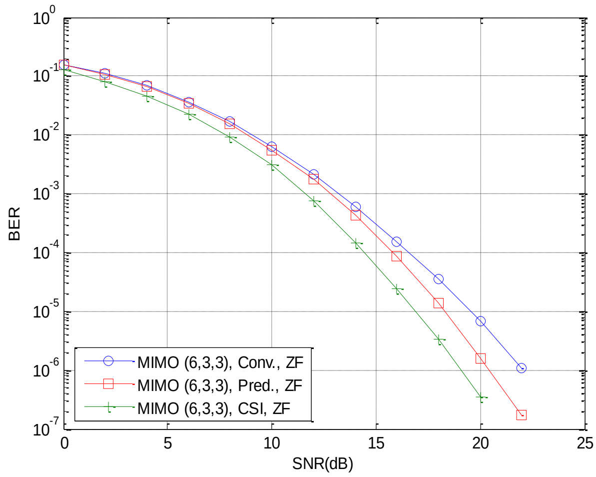

Fig. 6. BER comparison of conventional, CSI and prediction cases for a system with $(N_t, N_r, M) = (6,3,3)$, using 4-QAM, 4Bits feedback and ZF receiver.

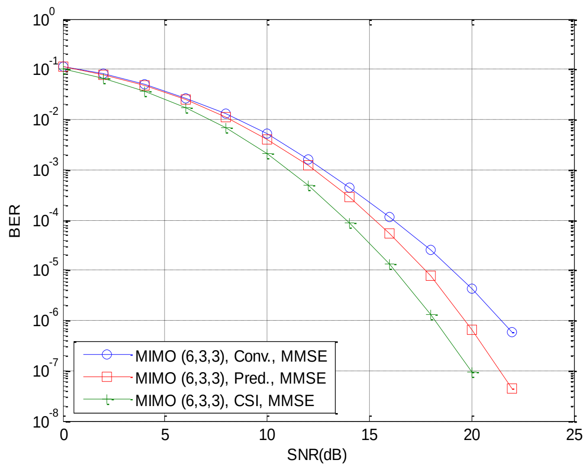

Fig. 7. BER comparison of conventional, CSI and prediction cases for a system with $(N_t, N_r, M) = (6,3,3)$, using 4-QAM, 4Bits feedback and MMSE receiver.

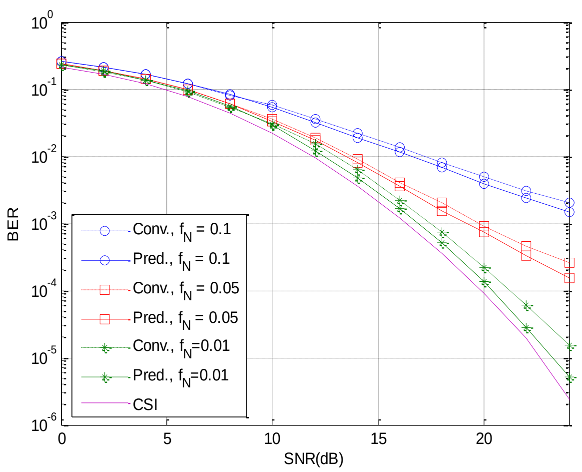

Fig. 8. BER comparison of conventional, CSI and prediction cases for a SM MIMO system with $(N_t, N_r, M) = (4,2,2)$ for different values of normalized Doppler frequency f_N.

Case 4: Finally, we investigate the BER performance for a 4×2 system for different values of the normalized Doppler frequency $f_N = f_d T_s$. Fig. 8 shows that, as f_N increases the BER performance degrades due to channel estimation error caused by a fast change of the channel. However, the proposed channel prediction demonstrates improved bit error rate performance over the conventional case for time-varying Rayleigh fading channels even for systems with high mobility.

IV. Conclusions

In this paper we assessed the performance of precoded spatial multiplexing MIMO systems in time-varying fading channels. A prediction method based on a Kalman filter has been proposed to overcome the dynamics of the channel, and mitigate the feedback delay effect. The prediction is made at the receiver based on the information that would be available for any spatial multiplexing MIMO system, and only the index of the selected optimal matrix is fed back to the transmitter. Therefore, the amount of the feedback information is the same as for the case when no precoder prediction is used. The effectiveness of this method was evaluated using computer simulation, and it is shown through the improved BER performance, that the proposed method mitigates the adverse time varying channel impairments, and reduces the feedback delay effects.

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