# Power Efficient Relay Selection and Diversity-Multiplexing Tradeoff in Two-way Relaying Networks 

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#### Abstract

Two-way amplify-and-forward (AF) relaying networks have been widely studied because of its simplicity and high spectral efficiency. In this paper, we investigate the relay selection problem and the diversity-multiplexing tradeoff (DMT) for two-way AF relaying systems over independent flat Rayleigh fading channels. We first derive the closed-form expression of the minimum total transmit power under the predefined data rate requirements. Based on the minimum total transmit power, we propose a simple relay selection which only requires user-relay channel state information (CSI). We proved that the proposed relay selection scheme can achieve the same DMT performance as obtained by the popular outage-optimal two-way opportunistic relaying scheme. Simulation results demonstrate that the proposed scheme can significantly decrease the power consumption compared with random relay selection and other schemes.


Index Terms-Two-way, amplify-and-forward, relay selection, power allocation

## I. Introduction

Recently, two-way relay network (TWRN) has drawn much attention due to its capability to mitigate the spectral efficiency loss of conventional one-way relaying system [1]. Both the amplify-and-forward (AF) and the decode-and-forward (DF) relaying protocols have been extended to TWRN [2], [3]. TWRN based on AF strategy (TWRN-AF) is of practical interest thanks to its low implementation complexity [4].

Two-way opportunistic relay (TWOR)-AF systems, a combination of two-way and opportunistic relaying, have been investigated in [5]-[9]. In view of the implementation of the TWOR-AF scheme, it can be accomplished in a centralized or distributed manner [5], [6]. In [5], a TWOR-AF scheme was implemented in a centralized manner, in which one of the two transceivers performs relay selection prior to two-way transmission, requiring continuous channel state information (CSI) feedback from all the links. However, numerous information exchanges will impair the system performance owing to causing congestion in the feedback

[^0]link as well as increasing overhead. In [6], a simple relay selection criterion based on the tight upper bounds of the end-to-end SNRs was proposed, which can be performed in a distributed manner with the requirement of local CSI, however, it did not consider the total power consumption at all the relays and terminals. In [7], an outage optimal opportunistic relaying was investigated based on the criterion of maximizing the worse received signal-tonoise ratio (SNR). In [8], a joint optimal power allocation and relay selection scheme that minimizes the symbol error probability was proposed. In [9], a joint relay selection and power allocation scheme based on maximizing the smaller of the received SNRs of the two transceivers under a total transmit power budget was proposed. All the above researches were operated to improve the system performance under the total transmit power constraints. However, in a battery-limited wireless relaying network, there is a close relationship between network lifetime and the power saving. Thus, it is essential to improve the power efficiency of wireless relaying system. In view of this, [10] proposed a relay selection strategy based on minimizing total power consumption while ensuring the transceivers' quality of service (QoS). But in the relay selection method of [10], the amplification factor was assumed to be a complex number according to the distributed beamforming algorithm given in [4], which adds to the complexity and the cost of computation. However, [10] has revealed that the solution of the power minimization problem does not depend on the phase of the beamforming weight in the scenario of single relay selection system, and therefore, no phase adjustment is required at the best relay. Motivated by all of the above, we develop a new power efficient relay selection strategy for TWRN-AF, where a best relay is selected based on the minimal power consumption criterion subject to constraints on the two transceivers' received SNRs. To sidestep the difficulty of the centralized implementation manner in the TWOR-AF scheme, we adopt a distributed TWOR-AF method only harnessing the user-relay CSI information, which can cut out numerous information exchanges caused by the centralized implementation. Unlike the relay selection algorithm discussed in [10], the amplification factor is assumed to be a real number in our work, thus can significantly reduce the implementation complexity required for obtaining the beamforming weight.

In this paper, we first derive the closed-form expression of power allocation by solving the optimization problem of total transmit power minimization. Then, based on the closed-form solution, we give a relay selection criterion which can be performed in a distributed manner and can be widely applied in practical TWRN. The relay which requires minimum total transmit power is selected out as the best relay. Since DMT is a useful performance measure for cooperative/relay systems, we analysis the DMT of our proposed relay selection scheme. The analytical results demonstrate that, under certain total power constraint, the proposed relay selection scheme can achieve the same DMT performance as obtained by the popular outageoptimal TWOR-AF scheme. Moreover, we use Monte Carlo simulations to validate the advantages of our scheme in power consumption.

Notations: Throughout this letter, $x \sim C N(a, b)$ denotes a complex Gaussian random variable with mean $a$ and variance $b$, and $|\cdot|$ represents the absolute value of a complex number.

The remainder of the paper is organized as follows. In Section II, the system and channel model are presented. Section III presents our power allocation and relay selection scheme based on total transmit power minimization. Section IV analyses the DMT performance of the proposed scheme. Simulation results are presented in Section V and conclusions are drawn in Section VI.


Fig. 1 System model.

## II. System and Channel Models

Consider a two-way relaying system illustrated in Fig.

1. Two terminals $S_{1}$ and $S_{2}$ exchange information with each other using one of the $N$ relay nodes, all channels undergo independent flat Rayleigh fading. We assume that all terminals are single-antenna devices and operate in a half-duplex mode, and there is no direct path between $S_{1}$ and $S_{2}$. We define $n_{1} \sim C N\left(0, \sigma_{1}^{2}\right)$ stands for the additive white Gaussian noise (AWGN) at $S_{1}$ and $n_{2} \sim C N\left(0, \sigma_{2}^{2}\right)$ stands for the AWGN at $S_{2}$, respectively. In the first time slot, $S_{1}$ and $S_{2}$ transmit their signals to all the relays simultaneously. Then, the received signal at relay $k(k=1, \cdots, N)$ can be expressed as

$$
\begin{equation*}
y_{k}=h_{1 k} \sqrt{P_{1}} x_{1}+h_{2 k} \sqrt{P_{2}} x_{2}+n_{R k} \tag{1}
\end{equation*}
$$

where $x_{1}$ and $x_{2}$ are the unit energy transmit signals from $S_{1}$ and $S_{2}$, respectively, $P_{1}$ and $P_{2}$ stand for the transmit power of $S_{1}$ and $S_{2}$, respectively,
$n_{R k} \sim C N\left(0, \sigma_{R k}^{2}\right)$ is the AWGN at relay $k$, and $h_{i k} \sim C N\left(0, \sigma_{i k}^{2}\right) \quad(i=1,2) \quad$ stands for the channel coefficient of $S_{i} \rightarrow k$. For the sake of simplicity, we assume that $\sigma_{R k}^{2}=\sigma_{1}^{2}=\sigma_{2}^{2}=N_{0}$. In the second time slot, the best relay (e.g. the $k$ th relay) broadcasts the received signal with the amplification factor given by

$$
\begin{equation*}
\alpha_{k}=\sqrt{P_{R k} /\left(P_{1} h_{k k}^{2}+P_{2} h_{2 k}^{2}+N_{0}\right)} \approx \sqrt{P_{R k} /\left(P_{1} h_{1 k}^{2}+P_{2} h_{2 k}^{2}\right)} \tag{2}
\end{equation*}
$$

where the approximation is get by ignoring the noise statistic at relay $k$ in conditions of high SNR, and $P_{p k}$ is the transmit power at relay $k$. Then, relay $k$ broadcasts signals to $S_{1}$ and $S_{2}$ and the receive signals at $S_{1}$ and $S_{2}$ can be written as

$$
\begin{align*}
y_{1} & =h_{k 1} \alpha_{k} y_{k}+n_{1} \\
& =h_{k 1} \alpha_{k}\left(h_{1 k} \sqrt{P_{1}} x_{1}+h_{2 k} \sqrt{P_{2}} x_{2}+n_{R k}\right)+n_{1}  \tag{3}\\
y_{2} & =h_{k 2} \alpha_{k} y_{k}+n_{2} \\
& =h_{k 2} \alpha_{k}\left(h_{1 k} \sqrt{P_{1}} x_{1}+h_{2 k} \sqrt{P_{2}} x_{2}+n_{R k}\right)+n_{2} \tag{4}
\end{align*}
$$

Assuming channel reciprocity, i.e. $h_{k k}=h_{k 1}$ and $h_{2 k}=h_{k 2}$, after self-interference cancellation [2], the receive signals at $S_{1}$ and $S_{2}$ can be rewritten as

$$
\begin{align*}
& y_{1}=h_{1 k} \alpha_{k}\left(h_{2 k} \sqrt{P_{2}} x_{2}+n_{R k}\right)+n_{1} \\
& y_{2}=h_{2 k} \alpha_{k}\left(h_{1 k} \sqrt{\left.P_{1} x_{1}+n_{R k}\right)+n_{2}}\right. \tag{5}
\end{align*}
$$

From (5), the end-to-end SNRs at $S_{1}$ and $S_{2}$ can be obtained as

$$
\begin{align*}
& \gamma_{1 k 2}=\frac{\alpha_{k}^{2}\left|h_{1 k}\right|^{2}\left|h_{2 k}\right|^{2} P_{1}}{N_{0}\left(1+\alpha_{k}^{2}\left|h_{2 k}\right|^{2}\right)}=\frac{\left|h_{k k}\right|^{2}\left|h_{2 k}\right|^{2} P_{1} P_{R k}}{N_{0}\left[\left|h_{2 k}\right|^{2}\left(P_{2}+P_{R k}\right)+\left|h_{k k}\right|^{2} P_{1}\right]}  \tag{6}\\
& \gamma_{2 k 1}=\frac{\alpha_{k}^{2}\left|h_{k k}\right|^{2} \mid h_{2 k}{ }^{2} P_{2}}{N_{0}\left(1+\left.\left.\left.\alpha_{k}^{2}\right|^{2}\right|_{k}\right|^{2}\right)}=\frac{\left|h_{1 k}\right|^{2}\left|h_{2 k}\right|^{2} P_{2} P_{R k}}{N_{0}\left[\left|h_{k k}\right|^{2}\left(P_{1}+P_{R k}\right)+\left|h_{2 k}\right|^{2} P_{2}\right]}
\end{align*}
$$

Accordingly, the mutual information of the links $S_{2} \rightarrow k \rightarrow S_{1}$ can be written as

$$
\begin{equation*}
I_{1}=\frac{1}{2} \log _{2}\left(1+\gamma_{2 k 1}\right) \tag{7}
\end{equation*}
$$

And the mutual information of the links $S_{1} \rightarrow k \rightarrow S_{2}$ can be written as

$$
\begin{equation*}
I_{2}=\frac{1}{2} \log _{2}\left(1+\gamma_{k 2}\right) \tag{8}
\end{equation*}
$$

## III. Power Efficient Relay Selection

## A. Power Allocation Basend on Total Transmit Power Minimization

For an arbitrary relay (e.g. the $k$ th relay), we aim to minimize the total power allocated to both source nodes and the relay while maintaining a lower bound for the end-to-end data rates, the constrained minimization problem can be formulated as follows

$$
\min _{P_{1}, P_{2}, P_{R k}} P_{1}+P_{2}+P_{R k} \quad \text { s.t. }\left\{\begin{array}{l}
(1 / 2) \log _{2}\left(1+\gamma_{2 k 1}\right) \geq R_{t h 1}  \tag{9}\\
(1 / 2) \log _{2}\left(1+\gamma_{1 k 2}\right) \geq R_{t h 2} \\
P_{1}, P_{2}, P_{R k} \geq 0
\end{array}\right.
$$

where $R_{t h 1}$ and $R_{t h 2}$ represent the minimum required date rates at the two terminals. After some algebraic manipulations, (9) can be rewritten as

$$
\min _{P_{1}, P_{2}, P_{R k}} P_{1}+P_{2}+P_{R k} \quad \text { s.t. }\left\{\begin{array}{l}
\frac{\alpha_{k}^{2}\left|h_{1 k}\right|^{2}\left|h_{2 k}\right|^{2} P_{1}}{N_{0}\left(1+\alpha_{k}^{2}\left|h_{2 k}\right|^{2}\right)} \geq \gamma_{t h 2}  \tag{10}\\
\frac{\alpha_{k}^{2}\left|h_{1 k}\right|^{2}\left|h_{2 k}\right|^{2} P_{2}}{N_{0}\left(1+\alpha_{k}^{2}\left|h_{1 k}\right|^{2}\right)} \geq \gamma_{t h 1} \\
P_{1}, P_{2}, P_{R k} \geq 0
\end{array}\right.
$$

where $\gamma_{t h 1}=2^{2 R_{t 1}}-1$ and $\gamma_{t h 2}=2^{2 R_{h 22}}-1$. It should be noted that the problem formulated in (10) is a convex problem and it should has a single global solution. Furthermore, it is interesting to note that the inequality constraints in (10) are satisfied with equality at the optimal solution. Otherwise, if, for example, the first constraint in (10) is satisfied with inequality at the optimal solution, then the optimal $P_{1}$ can be scaled down to satisfy this constraint with equality. This, however, further reduces the objective function in (10), thereby contradicting the optimality. Then, the problem of (10) can be reformulated as

$$
\begin{align*}
& \min _{P_{1}, P_{2}, P_{R k}} P_{1}+P_{2}+P_{R k} \\
& \text { s.t. } \frac{\left|h_{1 k}\right|^{2}\left|h_{2 k}\right|^{2} P_{1} P_{R k}}{N_{0}\left[\left|h_{2 k}\right|^{2}\left(P_{2}+P_{R k}\right)+\left|h_{1 k}\right|^{2} P_{1}\right]}=\gamma_{t h 2}  \tag{11}\\
& \frac{\left|h_{1 k}\right|^{2}\left|h_{2 k}\right|^{2} P_{2} P_{R k}}{N_{0}\left[\left|h_{1 k}\right|^{2}\left(P_{1}+P_{R k}\right)+\left|h_{2 k}\right|^{2} P_{2}\right]}=\gamma_{t h 1} \\
& P_{1}, P_{2}, P_{R k} \geq 0
\end{align*}
$$

Applying Karush-Kahn-Tucker (KKT) conditions for Lagrangian optimality, we can obtain the following optimal solution after some algebraic manipulations.

$$
\begin{align*}
& P_{1}=N_{0} \gamma_{2}\left(\frac{1}{\left|h_{1 k}\right|\left|h_{2 k}\right|}+\frac{1}{\left|h_{1 k}\right|^{2}}\right) \\
& P_{2}=N_{0} \gamma_{1}\left(\frac{1}{\left|h_{1 k}\right|\left|h_{2 k}\right|}+\frac{1}{\left|h_{22}\right|^{2}}\right)  \tag{12}\\
& P_{R k}=N_{0} \frac{\left(\left|h_{1 k}\right|+\left|h_{2 k}\right|\right)\left(\left|h_{1 k}\right| \gamma_{2}+\left|h_{2 k}\right| \gamma_{1}\right)}{\left|h_{1 k}\right|^{2}\left|h_{2 k}\right|^{2}}
\end{align*}
$$

Substituting (12) into (7) yields

$$
\begin{equation*}
\alpha_{k} \approx \sqrt{\frac{1}{\left|h_{1 k}\right|\left|h_{2 k}\right|}} \tag{13}
\end{equation*}
$$

According to (12), we can obtain the minimum total transmit power under the predefined data rate constraints when relay $k$ is selected.

$$
\begin{align*}
P_{T, k} & =P_{1}+P_{2}+P_{R k} \\
& =N_{0}\left(\gamma_{1}+\gamma_{2}\right) \frac{\left(\left|h_{1 k}\right|+\left|h_{2 k}\right|\right)^{2}}{\left|h_{1 k}\right|^{2}\left|h_{2 k}\right|^{2}} \tag{14}
\end{align*}
$$

## B. Proposed Relay Selection Criterion

In this paper, the best relay is chosen to minimize the total transmit power. Based on the closed-form solution of the minimum total transmit power shown in (14), the relay selection criterion can be described as

$$
\begin{align*}
b & =\arg \min _{k \in\{1,2, \cdots, N\}} P_{T, k} \\
& =\arg \min _{k \in\{1,2, \cdots, N\}} N_{0}\left(\gamma_{1}+\gamma_{2}\right) \frac{\left(\left|h_{1 k}\right|+\left|h_{2 k}\right|\right)^{2}}{\left|h_{1 k}\right|^{2}\left|h_{2 k}\right|^{2}} \tag{15}
\end{align*}
$$

Under the predefined $\gamma_{1}$ and $\gamma_{2}$, the result of (15) only relies on the local CSI between the selected relay and the two terminals, then the relay selection criterion can be simplified as

$$
\begin{align*}
b & =\arg \min _{k \in\{1,2, \cdots, N\}} \frac{\left(\left|h_{1 k}\right|+\left|h_{2 k}\right|\right)^{2}}{\left|h_{1 k}\right|^{2}\left|h_{2 k}\right|^{2}}  \tag{16}\\
& =\arg \min _{k \in\{1,2, \cdots, N\}}\left(\frac{1}{\left|h_{1 k}\right|}+\frac{1}{\left|h_{2 k}\right|}\right)
\end{align*}
$$

In the proposed relay selection scheme, only one relay is selected out of the N relays to forward signals in the second phase transmission. In order to reduce the cooperative/relay overhead among all the relays and the terminals, we propose a distributed timer technique which can be performed in a distributed manner without requiring global CSI at each relay, thereby reducing the implementation complexity and power consumption. The procedure of the relay selection scheme is conducted as follows.

Upon receiving signals from the two terminals, each relay then starts a timer $T_{k}$ whose duration is proportional to $\left(\frac{1}{\left|h_{\iota k}\right|}+\frac{1}{\left|h_{2 k}\right|}\right), k \in\{1,2, \cdots, \mathrm{~N}\}$. Note that each relay only needs to know the local CSI rather than the global CSI. According to (16), timer $T_{b}$ of the best relay $b$ will expire first. Then relay $b$ broadcasts its index to all the relay nodes over a control channel to signal its presence. It should be noted that the number of bits required for broadcasting is $\log _{2} N$. All the relays, while waiting for their timer to reduce to zero, are in listening mode. As soon as they hear another relay to flag its presence, they back off. After relay selection, the best relay $b$ will use its local channel state information to calculate $\alpha_{b}$, and then prepare signals to be broadcasted. Those relays, which have not been selected, will not participate in the relaying process.

## IV. DMT ANALYSIS

Since DMT is a fundamental performance measure to compare various cooperative diversity schemes in multirelay networks. We compare the DMT performance of our scheme with that of TWOR-AF scheme. We recall the definition of DMT from [11, equation (9.1) and (9.3)] as

$$
\begin{align*}
& r=\lim _{s n r \rightarrow \infty} \frac{R(s n r)}{\log s n r} \\
& d(r)=-\lim _{s n r \rightarrow \infty} \frac{\log \left[P_{\text {out }}(r \log s n r)\right]}{\log s n r} \tag{17}
\end{align*}
$$

where $s n r=P_{T, k} / N_{0}, d$ is the diversity gain, $r$ is the multiplexing gain, $P_{\text {out }}(\cdot)$ is the outage probability, and $R$ is the data rate.

The TWOR-AF system is said to be in outage if and only if either of the two sources is in outage. Thus, with the assumption that both two sources have the same transmission rate $R$, the outage event occurs when either $\gamma_{1 b 2}$ or $\gamma_{2 b 1}$ falls below the threshold $\gamma=2^{2 R}-1$, therefore, the outage probability can be formulated as

$$
\begin{align*}
P_{\text {out }} & =P_{r}\left\{\min \left(0.5 \log \left(1+\gamma_{1 b 2}\right), 0.5 \log \left(1+\gamma_{2 b 1}\right)\right)<R\right\}  \tag{18}\\
& =P_{r}\left\{\min \left(\gamma_{1 b 2}, \gamma_{2 b 1}\right)<\gamma\right\}
\end{align*}
$$

Substituting (12), (13) and (14) into (6) yields

$$
\begin{align*}
\gamma_{1 k 2} & =\frac{P_{T, k}}{N_{0}} \cdot \frac{\gamma_{2}\left|h_{1 k}\right|^{2}\left|h_{2 k}\right|^{2}}{\left(\gamma_{1}+\gamma_{2}\right)\left(\left|h_{1 k}\right|+\left|h_{2 k}\right|\right)^{2}}, k \in\{1,2, \cdots, \mathrm{~N}\}  \tag{19}\\
\gamma_{2 k 1} & =\frac{P_{T, k}}{N_{0}} \cdot \frac{\gamma_{1}\left|h_{1 k}\right|^{2}\left|h_{2 k}\right|^{2}}{\left(\gamma_{1}+\gamma_{2}\right)\left(\left|h_{1 k}\right|+\left|h_{2 k}\right|\right)^{2}}, k \in\{1,2, \cdots, \mathrm{~N}\}
\end{align*}
$$

For simplicity of exposition, we assume $\gamma_{1}=\gamma_{2}=\gamma$. Then we can obtain

$$
\begin{equation*}
\gamma_{1 k 2}=\gamma_{2 k 1}=\frac{P_{T, k}}{2 N_{0}} \cdot \frac{\left|h_{1 k}\right|^{2}\left|h_{2 k}\right|^{2}}{\left(\left|h_{1 k}\right|+\left|h_{2 k}\right|\right)^{2}}, \quad k \in\{1,2, \cdots, \mathrm{~N}\} \tag{20}
\end{equation*}
$$

Thus, (18) equal to

$$
\begin{equation*}
P_{\text {out }}=\left\{\gamma_{1 b 2}<\gamma\right\} \tag{21}
\end{equation*}
$$

In our relay selection scheme, the best relay is chosen to minimize the total transmit power under QoS constraints. In order to compare the DMT performance of our scheme with that of the TWOR-AF scheme under certain total transmit power constraint, after the best relay $b$ has been selected out, we increase the total power from $P_{T, b}$ to any one of $P_{T, k}(k \in\{1,2, \cdots, \mathrm{~N}\}, k \neq b)$. Since the total transmit power is increased, the end-to-end SNR will be greater than the threshold $\gamma$. From (19), it can be easily proved that $\gamma_{1 b 2}>\gamma_{1 k 2}=\gamma(k \in\{1,2, \cdots, \mathrm{~N}\}, k \neq b)$. Therefore, we can obtain

$$
\begin{align*}
& \gamma_{1 b 2}=\max _{k \in\{1,2, \cdots, \mathrm{~N}\}} \gamma_{1 k 2}  \tag{22}\\
& \text { s.t. } P_{T, b}=P_{T, k}(k \in\{1,2, \cdots, \mathrm{~N}\}, k \neq b)
\end{align*}
$$

Then, (22) equal to

$$
\begin{align*}
P_{\text {out }} & =P_{r}\left\{\max _{k \in\{1,2, \cdots, N\}} \gamma_{1 k 2}<\gamma\right\} \\
& =\prod_{k=1}^{N} P_{r}\left\{\gamma_{1 k 2}<\gamma\right\} \\
& =\prod_{k=1}^{N} P_{r}\left\{\left(\frac{1}{\left|h_{1 k}\right|}+\frac{1}{\left|h_{2 k}\right|}\right)>\sqrt{\frac{P_{T, k}}{2 N_{0} \gamma}}\right\}  \tag{23}\\
\text { s.t. } & P_{T, b}=P_{T, k}(k \in\{1,2, \cdots, \mathbf{N}\}, k \neq b)
\end{align*}
$$

Since $\left|h_{1 k}\right|$ and $\left|h_{2 k}\right|$ follow Rayleigh distributions with parameter $\sigma_{1 k}^{2}$ and $\sigma_{2 k}^{2}$, respectively, the probability density function (PDF) of $X=\frac{1}{\left|h_{t k}\right|}$ can be evaluated with the help of [12] to yield $p_{x}(x)=\left(2 /\left(\sigma_{1 k}^{2} x^{3}\right)\right) e^{-1 /\left(\sigma_{x}^{2} x^{2}\right)} U(x)$, and the PDF of $Y=\frac{1}{\left|h_{2 k}\right|}$ has a similar expression. Thus, the cumulative distribution function (CDF) of $Z=X+Y$ can be written as

$$
\begin{align*}
& P_{r}\{Z<\tau\}=\int_{0}^{\tau} \int_{0}^{z} \frac{2 e^{-\frac{1}{\sigma_{2 k}^{2} y^{2}}}}{\sigma_{2 k}^{2} y^{3}} \cdot \frac{\left.2 e^{\left(-\frac{1}{\sigma_{1 k}^{2}(z-y)^{2}}\right.}\right)}{\sigma_{1 k}^{2}(z-y)^{3}} d y d z \\
& =\int_{0}^{\tau} \frac{2 e^{-\frac{1}{\sigma_{2 k}^{2} y^{2}}}}{\sigma_{2 k}^{2} y^{3}} \cdot\left[\int_{y}^{\tau} \frac{2 e^{\left(-\frac{1}{\sigma_{1 k}^{2}(z-y)^{2}}\right)}}{\sigma_{1 k}^{2}(z-y)^{3}} d z\right] d y  \tag{24}\\
& =\int_{0}^{\tau} \frac{2}{\sigma_{2 k}^{2} y^{3}} e^{\left(-\frac{1}{\sigma_{2 k}^{2} y^{2}}-\frac{1}{\sigma_{1 k}^{2}(\tau-y)^{2}}\right)} d y
\end{align*}
$$

Using the variables above, (23) can be written as

$$
\begin{equation*}
P_{\text {out }}=\prod_{k=1}^{N}\left[1-P_{r}\left(Z<\sqrt{\frac{P_{T, k}}{2 N_{0} \gamma}}\right)\right] \tag{25}
\end{equation*}
$$

Substituting (24) to (25), we can get
$P_{\text {out }}=\prod_{k=1}^{N}\left[1-\int_{0}^{\sqrt{\frac{P_{T, k}}{2 N_{0} \gamma}}} \frac{2}{\sigma_{2 k}^{2} y^{3}} \times e^{-\frac{1}{\sigma_{2 k}^{2} y^{2}}-\frac{1}{\sigma_{1 k}^{2}\left(\sqrt{\frac{P_{T, k}}{2 N_{0} \gamma}}-y\right)^{2}}} d y\right]$
Approximated closed-form lower and upper bounds for (26) have been given by [13, equation (32)] as
where $K_{1}(\cdot)$ is the first order modified Bessel function. Note that each product term in (27) is obtained via a finite range integral of simple functions that is easily computed via a numerical computer package or calculator. Note also that the lower and upper bounds are in parallel, which suggests that the diversity gain of our scheme can
be obtained from the bounds. By applying the approximations $K_{1}(x) \approx 1-x$ and $e^{-x} \approx 1-x$ for $x \rightarrow 0$, at the high SNR region, (27) can be written as

$$
\begin{align*}
& P_{o u t}^{l b} \simeq\left(\frac{P_{T, k}}{N_{0}}\right)^{-N} \prod_{k=1}^{N}\left[2\left(\frac{1}{\sigma_{1 k}^{2}}+\frac{1}{\sigma_{2 k}^{2}}\right)\left(2^{2 R}-1\right)\right] \\
& P_{o u t}^{u b} \simeq\left(\frac{P_{T, k}}{N_{0}}\right)^{-N} \prod_{k=1}^{N}\left[4\left(\frac{1}{\sigma_{1 k}^{2}}+\frac{1}{\sigma_{2 k}^{2}}\right)\left(2^{2 R}-1\right)\right] \tag{28}
\end{align*}
$$

By substituting (28) into (17), we can obtain the DMT as

$$
\begin{align*}
d(r) & =-\lim _{\text {snr } \rightarrow \infty} \frac{\log \left(P_{\text {out }}\right)}{\log s n r} \\
& =-\lim _{s n r \rightarrow \infty} \frac{\log (s n r)^{-N}+N \cdot 2 r \log s n r}{\log s n r}  \tag{29}\\
& =N(1-2 r)
\end{align*}
$$

From (29), we know that the proposed relay selection scheme can achieve a maximal diversity gain of the number of relays and a maximal multiplexing gain of 0.5 under certain total power constraint, which is the same as TWOR-AF scheme.

## V. Simulation Results

In this section, we provide some numerical simulations to demonstrate the performance advantages of the proposed scheme. We consider a network consisting of 10 relays and two terminals. The channel coefficients $h_{1 k}$ and $h_{2 k}$ are generated as zero-mean complex Gaussian random variables with variance $\sigma_{1 k}^{2}$ and $\sigma_{2 k}^{2}$. In the numerical examples, we assume that (i) all channels are affected by the same fading environment following our model with path loss exponent $\alpha=4$, therefore, $\sigma_{i k}^{2}=d_{i k}^{-4}(\mathrm{i}=1,2)$, where $d_{i k}$ denotes the distance from $S_{i}(\mathrm{i}=1,2)$ to relay $k$, (ii) the locations of $S_{1}$ and $S_{2}$ are $(0,0)$ and $(1,0)$, respectively, (iii) the noise power at relays and at the two sources is 0 dBW , (iv) all relays are located in equidistant points from each other in a straight line from $\left(d_{1}, 0\right)$ to $\left(d_{1}, 1\right)$. The location and layout of all the relay nodes and terminals are shown in Fig. 2.


Fig. 2. location and layout of relay nodes and terminals.
Based on the minimum total transmit power criterion, a best relay is selected among the alternative 10 relays
according to the criterion given in (16). After relay selection, we can get the minimum total power according to (14). Besides, we assume that the simulation results in all figures are obtained using a Monte Carlo method with as many as 10000 samples.


Fig. 3. the average minimum total transmit power versus $\gamma_{2}$
Fig. 3 shows the average minimum total power versus data rate requirement $R_{t h 2}$. Another data rate requirement $R_{t h 1}$ is set as $R_{t h l}=1 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$ or $R_{t h 1}=5 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$. We assume that $d_{1}=0.5 \mathrm{~m}$. We compare the average minimum total power of our scheme with four different schemes: scheme 1 is random relay selection, scheme 2 refers to the relay selection scheme presented in [7], scheme 3 refers to the relay selection scheme proposed in [9], and scheme 4 refers to the multi-relay distributed beamforming method presented in [4] where all 10 relays are involved in relaying. As can be seen from this figure, the proposed scheme losses around 5 dBW in terms of total transmit power compared with scheme 4 , while it outperforms scheme 1 or scheme 2 by around 4 dBW or 1.5 dBW . Though scheme 4 has a better performance than the proposed scheme, the implementation of scheme 4 requires carrier-level synchronization and extra overhead, thus may be infeasible in complex and mobile networks. It can also be seen that there is a tiny gap between the proposed scheme and scheme 3, and the gap narrows as $\gamma_{2}$ close to $\gamma_{1}$ and widens as $\gamma_{2}$ far from $\gamma_{1}$. This is because the solution of scheme 3 relies on not only the local channel coefficients but also the SNR thresholds of the two terminals, while the solution of the proposed scheme only relies on local channel coefficients. Thus, our scheme is simpler in implementation than scheme 3.

Fig. 4 shows the average minimum total power versus the horizontal distance between $S_{1}$ and relays. We compare the average minimum total power of the proposed relay selection scheme with that of scheme 1 , scheme 2 , and scheme 3 . In this experiment, we assume that $R_{t 11}=R_{t 12}=2 b i t / s / \mathrm{Hz}$. It can be observed that, for all the schemes, the total power consumption increases when
the relays get closer to either of the two sources, this is due to the fact that in the case of unbalanced channels, the weaker channel will dominate the performance of the two-way relaying system and will consume more power to maintain the service qualities. It is clear from this figure that our relay selection scheme always consumes a lower total power compared with scheme 1 and scheme 2. Furthermore, this figure confirms that the total transmit power of our proposed scheme is the same as the total transmit power of scheme 3 when $R_{t t 1}=R_{t{ }^{2} 2}$, but our scheme can reduce the cooperative/relay overhead due to the distributed implementation manner. Consider of the power efficiency and the implementation complexity, our proposed relay selection scheme is more suitable for general wireless relaying systems.


Fig. 4. The average minimum total power versus $d$


Fig. 5. Optimum power allocation versus $d$
In Fig. 5, we plot the optimum power variations versus $d$ in two different scenarios: (i) $R_{t h 1}=R_{t h 2}=1 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$, (ii) $R_{t h 1}=0.5 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}, R_{t h 2}=1.5 \mathrm{bit} / \mathrm{s} / \mathrm{Hz}$. It can be seen that $P_{1}$ increases as $d$ changes from 0.1 to 0.9 while $P_{2}$ decreases, this is because the quality of the channel $S_{2} \rightarrow b$ improves as opposed to the channel $S_{1} \rightarrow b$. In the scenario (i), $P_{1}$ and $P_{2}$ are symmetrical with respect to
$d=0.5$, and $P_{R b}$ reachs a minimum value at $d=0.5$. It is worth remarking that $P_{R b}=P_{1}+P_{2}$ in this scenario, which means that half of the total power is allocated to the best relay and the two stages of information exchange are symmetric. In the scenario (ii), more quality of service is aimed at the terminal $S_{2}$. Hence the symmetry of $P_{1}$ and $P_{2}$ is obtained when $d<0.5$ and similarly $P_{R b}$ reaches minimum when $d>0.5$. Furthermore, in order to achieve better QoS at the terminal $S_{2}, P_{1}$ will dominate the total power when the quality of the channel $S_{1} \rightarrow b$ gets poor. In conclusion, the symmetry in power variations depends on the service quality requirements at the two terminals.

## VI. CONCLUSION

In this paper, a distributed power efficient relay selection scheme for TWRN-AF relaying network is proposed. The problem of power allocation which can minimize the total transmit power under date rate constraints is formulated and solved, and the best relay is chosen by comparing the minimum total powers among all the relays. We proved that proposed relay selection scheme can achieve a maximal diversity gain of the number of relays and a maximal multiplexing gain of 0.5 under certain total power constraint, which is the same as TWOR-AF scheme. Compared with the exiting relay selection schemes which are implemented in centralized manners, our scheme can reduce the cooperative/relay overhead owing to its distributed implementation. Simulation results show that the proposed scheme can significantly decrease the power consumption under the predefined data rate constraints. The proposed scheme is easy to implement and has the potential to find wide application in green communications.

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