Estimation of Time-Varying Communication Channels Using Reduced Length Chirp Pilots

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Abstract**—**In wireless communications, multipath fading causes significant performance degradation and necessitates channel estimation. The transmission of two consecutive chirps with different rates as a pilot sequence is a method that has been used for the estimation of linear time-varying (LTV) channel parameters. In this paper, we propose an improvement on the chirp-based channel estimation method for an LTV model. We show that a combination of a chirp with its complex conjugate, in particular a frequency-modulated (FM) sinusoid, provides us with an efficient pilot sequence. Besides reducing the length of the pilot sequence by half, the length and rate of our proposed pilot sequence can be adjusted to comply with *a priori* information on the channel. We implement the proposed method for an orthogonal frequency division multiplexing (OFDM) communication system and compare it with the conventional two-chirp method.

Index Terms**—**Linear time-varying channel model, conjugate chirps, orthogonal frequency division multiplexing.

I. INTRODUCTION

Wireless communication channels have undesirable effects on transmitted signals, such as attenuation, distortion, delays, and phase shifts. Therefore, estimating the time-varying response of a communication channel is crucial for improving the performance of a wireless communication system [1–3]. In this paper, we present a channel estimation method that uses the characteristics of chirps as eigenfunctions of linear time-varying (LTV) channel models, as well as the rate optimality of complex conjugate chirps [4, 5].

The application of chirp signals in parameter estimation of time-varying communication channels is not a new approach; chirps have also been used in sonar and radar systems [6–11]. Our aim here is to generate a more efficient channel estimation method than the one achieved by transmitting two consecutive chirps with different rates [4, 12, 13]. By sending two consecutive chirps with different rates as pilots and assuming that the channel does not change during this interval, the channel estimation problem can be converted into an estimation of harmonic frequencies in noise [12]. Additionally, as shown in [12], two complex conjugate chirps minimize the Cramér–Rao bound. Instead of sending two consecutive chirps, we propose a combination of a chirp and its complex conjugate, in particular a frequency-modulated (FM) sinusoid, as an efficient pilot sequence. As we showed in [5], using a linear chirp as input to an LTV channel, the linear chirp simplifies the model to that of a complex linear time-invariant (LTI) system with effective time shifts which are the combinations of the actual time shifts and Doppler frequency shifts. Using the eigenfunction property and time–frequency (TF) analysis, the frequency marginals of the dechirped received signal provided us with the information to estimate channel parameters. We performed a TF analysis in order to provide a justification for the use of FM sinusoids as pilots in the proposed method.

In Section II, we briefly review the model used for the multipath communication channel. In Section III, we show how to estimate the channel parameters using an FM sinusoid pilot. In Section IV, we illustrate the performance of our method for channel estimation in orthogonal frequency division multiplexing (OFDM) [14] and compare our results with those obtained using the two-chirp method, with conclusions following.

II. TIME-VARYING MULTIPATH CHANNEL MODEL

In mobile radio applications, because of the multipaths and relative motion between the transmitter and the receiver, which causes Doppler effects, the communication channel is typically modeled as an LTV system. In general, LTV channels are also known to be TF-dispersive or doubly dispersive. Their practical relevance made LTV models gain a lot of interest in the fields of signal processing, communication, information theory, and mathematics [15].

The time-varying frequency response of the channel characterizes the channel in terms of time delays, Doppler frequency shifts, and gains, all of which vary randomly in the modeling. An L-path fading channel with Doppler frequency shifts is generally modeled by a discrete-time separable impulse response [4, 16]:

$$h(n,m) = \sum_{l=0}^{L-1} h_l(n-m) f_l(n)$$

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$$=\sum_{l=0}^{L-1}\alpha_l\delta(n-N_l)e^{-jn\phi_l}, \quad (1)$$

where $h_l=\delta(n-N_l)$ as the impulse response of the all-pass systems corresponding to time delays $\{N_l\}$ and $f_l(n)=\alpha_l e^{-jn\phi_l}$. Doppler frequency shifts can be represented with $\{\phi_l\}$ and gains can be represented with $\{\alpha_l\}$. Although the model in Eq. (1) is assumed to be valid for the duration of a symbol, it changes with time and provides an approximation to the actual channel. Accordingly, the effect of the channel on the transmitted signal $s(n)$ is

$$y(n)=\sum_{l=0}^{L-1}\alpha_l s(n-N_l)e^{-jn\phi_l}, \quad (2)$$

where $y(n)$ is the channel output. Both deterministic and stochastic approaches are equally useful in describing a time-varying channel even though they are appealing for different aspects. The stochastic model is better suited for describing global behaviors, whereas the deterministic one is more useful in studying transmission through a specific channel realization [17]. As the channel effects cause the transmitted signal to disperse in time and frequency, coherent demodulation requires estimating the channel parameters.

III. The Proposed Chirp-Based Channel Estimation

Channel parameters can be estimated either by transmitting a pilot sequence or by blind estimation. We concentrate on pilot-based channel estimation where we use a chirp signal as our pilot sequence. As is well known, complex exponentials are the eigenfunctions of LTI systems, as they appear at the output of the system with amplitude and phase changed by the system. In [4, 5], it was shown that chirps are eigenfunctions of the LTV channel and chirps can enable us to model an LTV channel as LTI.

Let us start by defining the chirp signal $g(n)$ that we are going to use [18],

$$g(n)=e^{-\frac{j\pi}{8}}e^{j2\pi/N(0.5n^2)} \quad 0\le n\le N-1$$
$$=e^{j\theta n^2}, \quad (3)$$

as input to an LTV channel, where the instantaneous frequency $IF(n)=2\theta n=\frac{2\pi\tan(\beta)n}{N}$ and the chirp rate $\theta=\frac{\pi\tan(\beta)}{N}$ for $0<\beta\le\frac{\pi}{2}$. We can obtain the discrete Fourier transform (DFT) equivalent of $g(n)$ as $G(k)=e^{-\frac{j\pi}{8}}e^{j2\pi/N(0.5k^2)}, 0\le k\le N-1$. The initial and final instantaneous frequencies are 0 and $2\pi\tan(\beta)$, respectively. These DFT pairs $g(n)$ and $G(k)$ are also related in another way such that $G(k)=g(k)^*$, where * denotes a complex conjugate. As we showed in [5], these chirp signals have the following properties.

(1) A time delay N_0 on $g(n)$ corresponds to a frequency shift on $g(n)$ and a multiplication by a complex exponential depending on N_0 and the chirp rate:

$$g(n-N_0)=e^{-\frac{j\pi}{8}}e^{j2\pi(n-N_0)^2/2N}$$
$$=g(n)\,e^{-\frac{j2\pi N_0 n}{N}}\,e^{j\pi N_0{}^2/N},$$

which is also $g(n-N_0)=g(n)\,e^{-j2\theta N_0 n}e^{-j\theta N_0^2}$. In the equation above, $e^{-\frac{j2\pi N_0 n}{N}}$ corresponds to a Doppler shift of $\phi_0=2\pi N_0/N$ and $e^{j\pi N_0{}^2/N}$ is a constant.

(2) A frequency shift $\phi_1=2\pi N_1/N$ on $g(n)$ (i.e., multiplying $g(n)$ by $e^{-jn\phi_1}$) corresponds to an equivalent time delay $N_1=0.5\phi_1/\theta$ on $g(n)$ and a multiplication by a complex exponential depending on N_1 and the chirp rate:

$$g(n)e^{-jn\phi_1}=e^{-\frac{j\pi}{8}}e^{j2\pi/N(0.5n^2)}e^{-jn\phi_1}$$
$$=g(n-N_1)e^{-j\theta N_1^2},$$

where $g(n-N_1)$ shows a delay on $g(n)$ by N_1 samples and $e^{-j\theta N_1^2}$ is a constant.

(3) Using the properties obtained above, a time delay and a frequency shift at the same time on $g(n)$ correspond to an equivalent time shift $N_e=N_0+N_1$ on $g(n)$ and a multiplication by a complex exponential:

$$g(n-N_0)e^{-jn\phi_1}=g(n)e^{-\frac{j2\pi(N_0-N_1)}{N}}e^{\frac{j\pi N_0{}^2}{N}}$$
$$=g(n-N_e)\,e^{\frac{j\pi(N_1{}^2-2N_0N_1)}{N}}.$$

Generalizing Eq. (1), the output of the channel corresponding to the input $s(n)=g(n)$ can be derived as follows:

$$y(n)=\sum_{l=0}^{L-1}\alpha_l e^{j\theta(N_l{}^2-N_{el}^2)}g(n-N_{el}), \quad (4)$$

where $\{N_{el}=N_l+\phi_l/2\theta\}$ are referred to as equivalent time delays $\{N_l\}$ depending on the actual time delay and the Doppler frequency shifts $\{\phi_l\}$ in each path.

A. FM Sinusoids for Channel Estimation

The channel output corresponding to Eq. (4) indicates that it is possible to model the time-varying channel as a complex LTI system. The effect of the channel on the input chirp can be visualized in the TF domain as that of delaying the chirp in time by N_l samples and then shifting the resulting chirp in the frequency axis by ϕ_l radians, which correspond to an equivalent time shift of N_{el}samples. Accordingly, by dechirping $y(n)$ with $g^*(n)$, we obtain N_{el} values. However, this will not be enough to find the actual time and the Doppler frequency shifts, but two chirps with opposite rates, θ and $-\theta$, are necessary [12]. In this paper, we use the properties explained in [5] for rewriting the channel output $y(n)$ in Eq. (4) and obtain an eigenfunction relation in terms of the chirp $g(n)$. We rewrite Eq. (4) according to the properties given above:

$$y(n)=\sum_{l=0}^{L-1}g(n)\alpha_l e^{j\theta N_l{}^2}e^{-j2\theta N_{el}n},$$

$$g(n)\sum_{l=0}^{L-1}\alpha_l e^{jIF(0.5\,N_l^2)}e^{-jIF(N_{el})n}, \quad (5)$$

where $g(n)$ acts as the eigenfunction of the LTV model of the channel.

The response of the channel occurs at instantaneous frequencies, $IF(N_{el})$ and $IF(0.5\,N_l^2)$, with $e^{-jIF(N_{el})n}$ corresponding to the equivalent shift in time. Let us now consider the use of $g(n)$ and $g^*(n)$ as pilots for channel estimation [12] (as suggested in [12], we take θ and $-\theta$ as the chirp rates), but instead of sending two consecutive chirps as pilots, we propose to use a combination of these chirps, for instance, an FM sinusoid:

$$g_T(n) = g(n) + g^*(n) = 2\cos(\theta n^2), \quad (6)$$

where Wigner–Ville TF representations are as shown in Fig. 1. With this combination as input, the channel output due to the linearity of the model is

$$y(n) = g(n)\left[\sum_{l=0}^{L-1}\alpha_l e^{j\theta\,N_l^2}e^{-j2\theta N_{el}n}\right]$$

$$+\quad g(n)^*\left[\sum_{l=0}^{L-1}\alpha_l e^{-j\theta\,N_l^2}e^{j2\theta N_{el}n}\right]$$

$$= g(n)f_1(n) + \; g(n)^* f_2(n), \quad (7)$$

where $f_1(n)$ and $f_2(n)$ are the terms in the brackets. From Eq. (4), the equivalent delays are as follows:

$$N_{el}^{(1)} = N_l + \frac{\phi_l}{2\theta},$$

$$N_{el}^{(2)} = N_l - \frac{\phi_l}{2\theta}, \quad (8)$$

which can be used to calculate the actual time delays $\{N_l\}$ and the Doppler frequency shifts $\{\phi_l\}$.

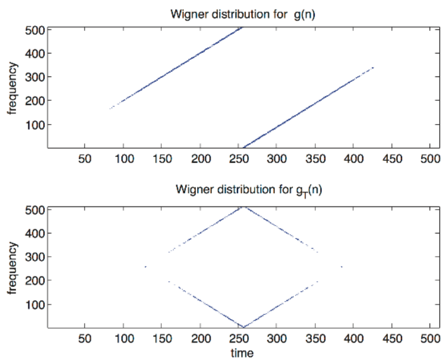

Fig. 1. Wigner distributions for chirps and FM sinusoids.

Figure 2 shows the channel output for a four-path channel with different Doppler frequencies as multiples of $\frac{2\pi}{N}$ and delays of [40 80 120 160] samples. Dechirping the output $y_T(n)$ by $g(n)^*$ and $g(n)$, we get $h_1(n) = y_T(n)\,g(n)^*$ and $h_2(n) = y_T(n)\,g(n)$ or

$$h_1(n) = f_1(n) + \; e^{-j2\theta n^2}f_2(n),$$
$$h_2(n) = f_2(n) + \; e^{j2\theta n^2}f_1(n). \quad (9)$$

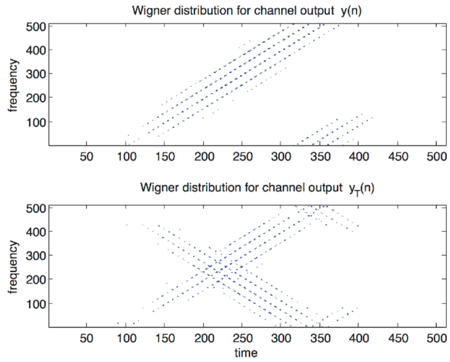

Fig. 2. Wigner distribution for four-path channel output.

B. TF Approach

Using a TF distribution that localizes chirps well, such as the Wigner distribution [19], the corresponding frequency marginals of $h_1(n)$ and $h_2(n)$ (see Fig. 3) will provide the information needed to obtain the channel parameters. Indeed, these marginals are related to the Fourier transform of $h_1(n)$ and $h_2(n)$ as shown in the following.

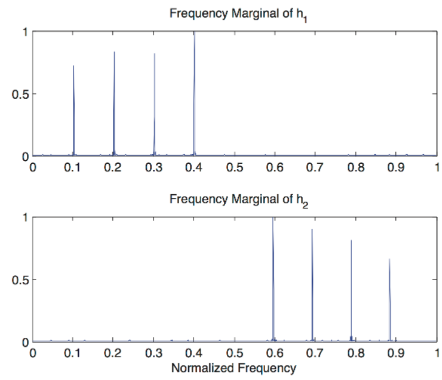

Fig. 3. Frequency marginals obtained by Wigner distribution for a four-path channel.

Thus, the connection of the frequencies where the peaks occur in Fig. 3 with the parameters can be shown by computing the Fourier transform of $h_1(n)$ and $h_2(n)$ as well. To do so, we consider the Fourier transform of a signal $f(n)$ multiplied by a chirp $c(n)$, with Fourier transform $C(\omega)$, as shown in Fig. 4.

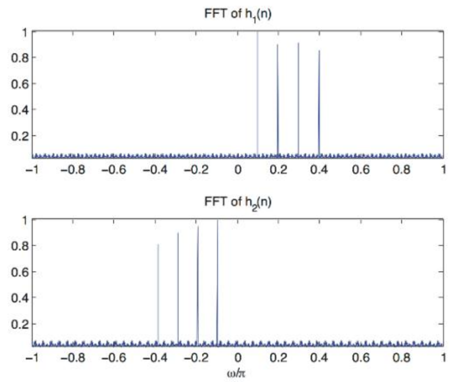

Fig. 4. Spectrum for a four-path channel.

Representing $f(n)$ by its inverse transform, we have

$$FT[f(n)c(n)] = \sum F(k)C(\omega - \omega_k), \quad (10)$$

which shows a flat spectrum if the chirp is broadband. The Fourier transforms of $f_1(n)$ and $f_2(n)$ are given by

$$F_1(\omega) = 2\pi \sum_{l=0}^{L-1} \alpha_l e^{j\theta N_l{}^2} \delta\left(\omega - 2\theta N_{el}^{(1)}\right), \quad (11)$$

$$F_2(\omega) = 2\pi \sum_{l=0}^{L-1} \alpha_l^* e^{-j\theta N_l{}^2} \delta(\omega - 2\phi_l - 2\theta N_{el}^{(1)}).$$

On the other hand, the Fourier transforms of $h_1(n)$ and $h_2(n)$ are composed of a wide-band low-amplitude part and impulses at frequencies $\{\omega_l = 2\phi_l\}$ for $F_1(\omega)$ and $\{\omega_l = 2\phi_l + 2\theta N_{el}^{(1)}\}$ for $F_2(\omega)$. Once we obtain $N_{el}^{(1)}$ from the first equation, we will use the second equation to obtain Doppler shifts ϕ_l, after which we can find the actual time delays N_l using the definition of $N_{el}^{(1)}$ in Eq. (8). The number of peaks is an estimate of L, and an estimate of the attenuations can be found by looking at the amplitudes of the peaks. Indeed, the estimates of α_l can be found from

$$H\left(2\theta N_{el}^{(1)}\right) \approx 2\pi\tilde{\alpha}_l e^{-j\theta N_l{}^2}. \quad (12)$$

In the above derivations, we assumed that no noise was present. When using the received signal,

$$r(n) = y_T(n) + \eta(n), \quad (13)$$

where $\eta(n)$ is the channel noise, we need to use the dechirped signals $r(n)\, g(n)^*$ and $r(n)\, g(n)$. Again, the Wigner distribution can be used to find the frequency marginals of the dechirped signals, or we can use the periodogram [20] to estimate the frequencies where the peaks of the spectra of $r(n)\, g(n)^*$ and $r(n)\, g(n)$ occur.

In the discrete implementation of the method, we need to consider the significant difference in scale between the time and the Doppler frequency shifts. Given as *a priori* information, the possible range of values for the time and frequency delays, the length of the FM sinusoid, N, and the rate, θ, can be adjusted. After choosing the value of N to represent the range of time delays, one can then choose the chirp rate $\theta = \frac{\pi \tan(\beta)}{N}$ by letting the angle β be such that the instantaneous frequency goes from 0 to $\phi_{\max}$, the maximum value of the Doppler shift expected. Thus, the resolution is set appropriately for both the time and the frequency delays.

IV. The Proposed Chirp-Based Channel Estimation

We implement the transmission of 1,024 symbols in each OFDM block. The data symbols are QPSK-modulated. The FM cosine and two-chirp pilots are used for channel estimation; in each case, pilot sequences are sent every six symbols. Letting the bandwidth be $BW = B$ kHz, if we wish to transmit M bits/frame, the center frequencies of the subchannels will be B/M kHz, $k = 0, \ldots, M-1$. The model that we use in our simulation for the communication channel is a noisy, frequency-selective fading channel, valid for the duration of an OFDM frame, and has L = 4 paths, each with attenuation α_l and time and frequency shifts τ_l and Φ_l, respectively. For the discrete-time model, it is assumed that the sampling frequency rate F_s is chosen appropriately so that the time shifts are $\tau_l = N_l T_s$ and likewise the Doppler frequency shifts are $\Phi_l = \phi_l F_s$ for some integers $\{N_l\}$.

We assumed that the relative velocity between the transmitter and the receiver is between 0 and 150 km/h so that the Doppler frequency shifts vary between 0 and 150 Hz, and we used a sampling frequency $F_s = 2B$, where the available bandwidth B = 30 kHz. As expected, in Fig. 5, the BER is larger for the case in which each path is assigned a different frequency shift, compared with the case where the Doppler frequency shift is the same for each path. The BERs obtained using two-chirp pilots and the proposed FM sinusoid for the channel estimation method are very close to each other, using just half the length of the two-chirp pilot sequence.

V. Conclusions

In this paper, we proposed an improvement on chirp channel estimation. The pilot signal in our system is an FM sinusoid obtained by combining a linear chirp with its conjugate, which allows an accurate estimation of the channel parameters needed in developing coherent detectors without the need for sending two consecutive chirps during a two-symbol duration. We were able to estimate channel parameters by sending an FM sinusoid of one-symbol duration. Our method is justified using TF analysis and showing that linear chirps have eigenfunction properties. We simulated an OFDM system and compared our method with the two-chirp method, showing a very similar performance to that of the two-chirp method without sending two consecutive chirps. In our future work, we will explore other channel models such as the Basis Expansion Model (BEM) using the proposed estimation method with an increased number of multipaths.

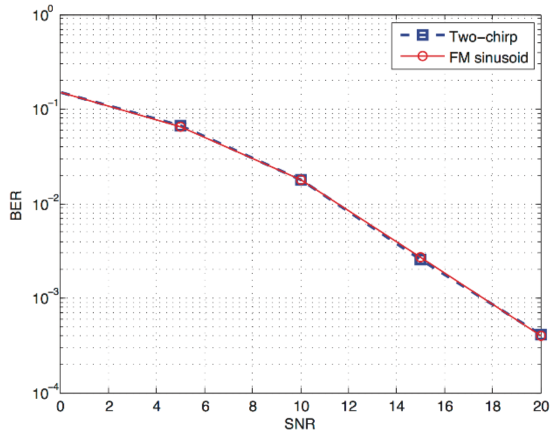

Fig. 5. BER versus SNR for chirp channel estimation in OFDM using 1,024 symbols.

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