Coverage Analysis for Multiuser MIMO Broadcast Systems

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***Abstract*— In this paper, we present the closed-form expressions for the link outage and coverage of the zero-forcing beamforming (ZFB) and zero-forcing dirty-paper coding (ZF-DPC) multiuser multi-input multi-output (MIMO) broadcast systems. We find that the ZF-DPC MIMO broadcast system has a larger diversity order and better coverage compared with the ZFB MIMO broadcast system. Furthermore, it is observed that the ZFB MIMO broadcast system with round-robin scheduling has only the diversity order of one and its cell coverage can only approach to that of the weakest link of the ZF-DPC MIMO broadcast system. By selecting the best group of users, multiuser scheduling can function as a *soft* coverage enhancement technique without increasing the extra transmission power in the physical layer. Our analytical formula can estimate to what extent the coverage performance of the ZF-DPC MIMO broadcast system can be improved as the number of users increases. Hence, the effect of increasing the number of antennas on the coverage performance of the ZF-DPC MIMO broadcast system can be quantitatively analyzed subject to the same transmission power from each base station.**

***Index Terms*— MIMO systems, zero-forcing beamforming, zero-forcing dirty-paper coding, coverage, MIMO broadcast channels, outage probability, diversity order.**

I. Introduction

Multiple-input multiple-output (MIMO) systems can significantly increase spectral efficiency by exploiting the degree of freedom in the spatial domain created by multiple antennas. In the point-to-multipoint multiuser MIMO broadcast channels, even with only one single receive antenna at the user end, the spatial multiplexing gain can be also achieved by sending precoded data across multiple transmit antennas to a group of users simultaneously [1]. With complete channel state information (CSI) available at the transmitter, the maximum sum rate of MIMO broadcast systems can be achieved by dirty paper coding (DPC) [1]. However, computational complexity and the requirement of full CSI at the transmitter limit the applicability of DPC.

In the literature, the new lines of research from multiuser MIMO broadcast systems are classified into three categories:

- First, rather than using the optimal DPC, the suboptimal but more practical MIMO broadcast schemes were proposed [2]–[6], such as the zero-forcing dirty-paper coding (ZF-DPC), zero-forcing beamforming (ZFB), block-diagonalization (BD), orthogonal random beamforming, and receive ZF beamforming [7]–[10]. These suboptimal schemes can asymptotically achieve the same throughput of DPC when the number of users approaches to the infinity.
- Second, another important research direction for MIMO broadcast systems is to investigate the impacts of limited CSI due to the finite-rate or erroneous CSI feedback [11]–[17]. For example, [12] showed that the feedback load per user must be scaled together with both the number of transmit antennas as well as the system SNR to achieve the full multiplexing gain with the near-perfect CSI.
- Third, MIMO broadcast transmission strategies were also applied to the multi-cellular scenario to cancel the inter-cell interference for improving spectral efficiency [18]–[20]. For example, the network base station (BS) coordination conception is proposed based on ZFB and ZF-DPC schemes in [19], [20]. In This concept is also called the coordinated multipoint (CoMP) transmission in the third Generation Partnership Project (3GPP) Long-Term Evolution-Advanced (LTE-A) and the collaborative MIMO (Co-MIMO) transmission in the IEEE 802.16m Worldwide Interoperability for Microwave Access (WiMAX), respectively [21]–[23].

Although the capacity issues of MIMO broadcast systems have been extensively investigated, to our knowledge, the studies on the coverage performance of MIMO broadcast systems are rarely seen in the literature. From the perspective of tradeoff between multiplexing and diversity for a MIMO system [24], the transmit MIMO broadcast systems may be a diversity-deficient scheme due to it realizes spatial-multiplexing personalized transmissions. In this paper, we derive the analytical closed-form expressions for the link outage probability, link diversity order,

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and the reliable coverage radius of a multiuser MIMO broadcast system. It is found that multiuser scheduling can function as a **link diversity compensation and soft coverage extension** technique for the MIMO broadcast systems. The concept of soft coverage extension by multiuser scheduling was suggested for point-to-point MIMO systems in [25] since multiuser scheduling can improve the coverage of MIMO broadcast systems without increasing transmission power in the physical layer. In [26], the authors derived the closed-form expressions for the outage and error rate performances in a single-user ZF-MIMO system with imperfect channel feedback. However, both [25], [26] only considered the single-user MIMO system that serves one single user at any time instant, rather than the MIMO broadcast systems that serves a group of users simultaneously. Our developed analytical framework can evaluate to what extent the multiuser scheduling can improve the coverage performance of the multiuser MIMO broadcast systems.

In this paper, we focus on two famous precoding schemes, ZF-DPC and ZFB, to address the link and coverage performance of a MIMO broadcast system. Different from the results in [27], we have further provided some other works to make the link performance issue of MIMO broadcast systems more complete. The additional works can be summarized as follows:

- We derived the link diversity order of MIMO broadcast systems with ZF-DPC and ZFB transmission precoding schemes. We find that the diversity order of the i-th link for an N_t-link ZF-DPC broadcast system without scheduling and ordering is $N_t - i + 1$ and that of the ZFB broadcast system without scheduling is one, where N_t is the number of transmit antennas. Taking the strongest link of ZF-DPC as an example. The diversity order becomes KN_t instead of N_t when combining with K-user greedy scheduling. In [27], we have observed this phenomenon via simulations but not provided theoretical proofs.
- We consider the coverage extension issue with different number of users K and antennas N_t when taking advantage of multiuser scheduling. We demonstrate a *soft* coverage enhancement technique for MIMO broadcast systems by taking advantage of multiuser scheduling.

The rest of this paper is organized as follows. Section II introduces the ZF-DPC and ZFB MIMO broadcast systems. In Section III, we define the link outage probability, diversity order, and reliable coverage radius in MIMO broadcast systems. In Sections IV and V, we derive the analytical expressions of these coverage related performance metrics for the ZF-DPC and ZFB MIMO broadcast systems. Numerical results are shown in Section VI. We give our concluding remarks in Section VII.

II. Background

A. System Model

Consider a single-cell multiuser MIMO broadcast system with a BS and K users. The BS is equipped with N_t transmit antennas, but each of K user terminals has only one receive antenna. Thus, N_t users are selected from K users for simultaneous transmission with different data streams. The subset of users' indices to which a BS intends to transmit different information is denoted by $\mathcal{S} \subset \{1, \ldots, K\}$, $|\mathcal{S}| = N_t$.

The beamforming weight matrix at the transmitter is denoted by $\mathbf{W} = [\mathbf{w}_1 \ \ldots \mathbf{w}_{N_t}]$, where $\mathbf{w}_i \in \mathbb{C}^{N_t \times 1}$ and the input signal vector is denoted by $\mathbf{u} = [\sqrt{P_1}u_1, \ldots, \sqrt{P_{N_t}}u_{N_t}]^T$. Here u_i and P_i represent the uncorrelated unit-power signal symbol and the power of the symbol associated with user i, respectively. Then, the transmitted signal vector $\mathbf{x}$ is written as $\mathbf{x} = \mathbf{Wu} = \sum_{i=1}^{N_t} \sqrt{P_i}\mathbf{w}_i u_i \in \mathbb{C}^{N_t \times 1}$. Let $\mathbf{y} \in \mathbb{C}^{N_t \times 1}$ be the received signal vector, and $\mathbf{G}(\mathcal{S})$ be the $N_t \times N_t$ channel matrix corresponding to $\mathcal{S}$. Denote $\mathbf{n} \in \mathbb{C}^{N_t \times 1}$ as the complex Gaussian noise vector with $\mathrm{E}[\mathbf{nn}^H] = \sigma^2 \mathbf{I}_{N_t}$, where $(\cdot)^H$ denotes conjugate transpose. Then, the received signal can be expressed as

$$\mathbf{y} = \mathbf{G}(\mathcal{S})\mathbf{x} + \mathbf{n} = \mathbf{gH}(\mathcal{S})\mathbf{x} + \mathbf{n} \ , \tag{1}$$

where $\mathbf{g}$ is an $N_t \times N_t$ diagonal matrix with $\mathbf{g} = \mathrm{diag}\{\sqrt{g_1}, \sqrt{g_2}, \ldots, \sqrt{g_{N_t}}\}$ and g_i depicts the path loss between the BS and user i. For a user at a distance of R from the BS, g_i can be written as [28]

$$10 \log_{10} g_i = -10\mu \log_{10} R + g_0 \quad [\mathrm{dB}] \ , \tag{2}$$

where μ is the path loss exponent and g_0 is a constant subject to certain path loss models [1]. Assume that all users experience independent flat Rayleigh fading and the transmission power is constrained by $\mathrm{E}[\mathbf{x}^H\mathbf{x}] = P_T$.

B. Zero-Forcing Dirty-Paper Coding (ZF-DPC)

Based on QR-type decomposition, a suboptimal solution of $\mathbf{W}$ was found in [1]. Let $\mathbf{H}(\mathcal{S}) = \mathbf{LQ}$ be the QR-type decomposition of $\mathbf{H}(\mathcal{S})$, where $\mathbf{L}$ is a lower triangular matrix and $\mathbf{Q}$ is a unitary matrix. With $\mathbf{W} = \mathbf{Q}^H$, the corresponding system model in (1) can be written as

$$y_i = l_{i,i}\sqrt{g_i P_i}u_i + \sum_{j<i} l_{i,j}\sqrt{g_j P_j}u_j + n_i \ , \quad i = 1, \ldots, N_t. \tag{3}$$

Note that $\mathbf{W} = \mathbf{Q}^H$ can cancel the interference from users with indices $j > i$. The remaining interference terms with indices $j < i$ are taken care of by applying DPC successively. For simplicity, we consider the equal power allocation, that is, $P_i = P_T/N_t$, where $i = 1, \ldots, N_t$. The rate of the ith link for ZF-DPC is $\log_2(1 + |l_{i,i}|^2\rho_i/N_t) = \log_2(1+\gamma_i)$, where ρ_i is the average received signal-to-noise ratio (SNR), γ_i is the effective received SNR, and the term

[1] For example, in a macro-cell environment, the path loss model of modified COST-231 Hata urban and suburban models are respectively $-35\log_{10} R - 31.5$ [dB] (i.e., $\mu = 3.5$ and $g_0 = -31.5$ dB) and $-35\log_{10} R - 34.5$ [dB] (i.e., $\mu = 3.5$ and $g_0 = -34.5$ dB) [29], where the antenna height of the base station is 32 m, the antenna height of the user terminal is 1.5 m, and carrier frequency is 1.9 GHz.

$|l_{i,i}|^2$ can be viewed as the effective channel gain at the ith link. Specifically, ρ_i can be represented as

$$\rho_i = \frac{P_T g_i}{\sigma^2} = \frac{P_T 10^{g_0/10}}{\sigma^2 R_i^{\mu}} \quad . \tag{4}$$

C. Zero-Forcing Beamforming (ZFB)

The ZFB scheme [1] aims at inverting the channel matrix to create orthogonal channels between the transmitter and the receiver. By choosing the weight matrix $\mathbf{W} = \mathbf{H}(\mathcal{S})^H \left(\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^H\right)^{-1}$, the corresponding system model in (1) can be written as

$$\mathbf{y} = \mathbf{g}\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^H(\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^H)^{-1}\mathbf{u} + \mathbf{n} = \mathbf{g}\mathbf{u} + \mathbf{n} \ , \tag{5}$$

and the ith received signal is given by $y_i = \sqrt{g_i P_i} u_i + n_i$. Due to the transmit power constraint $E[\mathbf{x}^H \mathbf{x}] \leq P_T$, we have the following relation:

$$\|\mathbf{w}_1\|^2 P_1 + \ldots + \|\mathbf{w}_{N_t}\|^2 P_{N_t} \leq P_T \ , \tag{6}$$

where $\|\mathbf{w}_i\|^2 = [(\mathbf{H}(\mathcal{S})\mathbf{H}(\mathcal{S})^H)^{-1}]_{i,i}$. In (6), it implies that ZFB incurs an excess transmission power penalty due to the required interference cancellation power on $\mathbf{W}$. According to (6), power loading is $\|\mathbf{w}_i\|^2 P_i = P_T/N_t$, where $i = 1, \ldots, N_t$. As a result, the data rate at the ith link of ZFB is

$$\log_2\left(1 + \frac{g_i P_i}{\sigma^2}\right) = \log_2\left(1 + \frac{b_i \rho_i}{N_t}\right) = \log_2(1 + \gamma_i) \ , \tag{7}$$

where $b_i = 1/\|\mathbf{w}_i\|^2$ is the effective channel gain.

III. Performance Metrics

A. Link Outage Probability

We first define the link outage probability to reflect what extent a MIMO broadcast system can reliably support the corresponding link quality. For a single-input single-output system (SISO), link outage [30] is usually defined as the probability of the effective received SNR is less than a predetermined value γ_{th}, i.e., $P_{\text{out}} = P_r\{\gamma < \gamma_{th}\}$. As for the MIMO broadcast systems, all the data links serve different individual users. Thus, we can define the link outage probability of the ith link the same as in the SISO case, i.e., $P_{\text{out}}^i = P_r\{\gamma_i < \gamma_{th}\}$.

B. Diversity Order

Let link outage probability $P_{\text{out}}(\cdot)$ be a function of SNR. Then, the link diversity order D_{order} is defined as [31]

$$D_{\text{order}} \triangleq -\lim_{\rho \to \infty} \frac{\log P_{\text{out}}(\rho)}{\log \rho} \quad , \tag{8}$$

where ρ is the receive SNR. The metric can provide an intuitional observation on link performance.

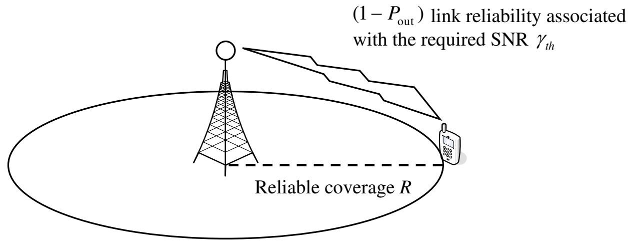

Fig. 1. Illustration of link reliability and reliable coverage.

C. Link Coverage Reliability

Referring to the link outage probability P_{out}, we further define $(1 - P_{\text{out}})$ as the link coverage reliability for its corresponding link radius associated with the required SNR as shown in Fig. 1. Specifically, the link coverage reliability $(1 - P_{\text{out}})$ represents the probability of the effective received SNR being higher than γ_{th}. Therefore, the link radius associated with the required SNR and $(1 - P_{\text{out}})$ reliability is defined as the **reliable coverage**. Typically, 90% link reliability is required for most wireless systems.

IV. Analysis of MIMO Broadcast Systems Without Scheduling

A. ZF-DPC without Scheduling

At first we analyze the coverage performance of ZF-DPC MIMO broadcast systems without user selection, i.e., based on round-robin scheduling. Clearly, selecting users randomly cannot result in multiuser diversity gain. By Lemma 2 in [1], $d_i = |l_{i,i}|^2$ is independent central Chi-squared random variable $\mathcal{X}^2_{2(N_t - i + 1)}$ with $2(N_t - i + 1)$ degrees of freedom. The probability density function (PDF) of a Chi-squared random variable $\mathcal{X}^2_{2a}$ is $f(z) = z^{a-1}e^{-z}/(a-1)!$ for $z > 0$. Thus, the PDF of the effective channel gain d_i can be written as

$$f_{d_i}(z) = \frac{z^{N_t - i} e^{-z}}{(N_t - i)!} \quad \text{for} \quad i = 1, \ldots, N_t \ , \tag{9}$$

where $i = 1$ and $i = N_t$ represent the strongest and the weakest links in the statistics, respectively. The corresponding cumulative distribution function (CDF) of d_i can be written as

$$F_{d_i}(z) = 1 - \frac{\Gamma(N_t - i + 1, z)}{\Gamma(N_t - i + 1)} = 1 - \Gamma_{\text{R}}\left(N_t - i + 1, z\right) \ , \tag{10}$$

where $\Gamma(a) = \int_0^\infty t^{a-1} e^{-t} dt$ is the complete gamma function, $\Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt$ is the upper incomplete gamma function, and $\Gamma_{\text{R}}(a, x) = \frac{\Gamma(a,x)}{\Gamma(a)}$ is the regularized gamma function. The CDF of the effective received SNR $\gamma_i = d_i \rho_i / N_t$ for the ith link is

$$F_{\gamma_i}(\gamma) = F_{d_i}\left(\frac{N_t \gamma}{\rho_i}\right) = 1 - \Gamma_{\text{R}}\left(N_t - i + 1, \frac{N_t \gamma}{\rho_i}\right) \quad . \tag{11}$$

Thus, for a given threshold $\gamma_{th} > 0$, the link outage probability for the ith link of the ZF-DPC MIMO broadcast system is

$$P^i_{\text{out}} = F_{\gamma_i}(\gamma_{th}) = \quad 1 - \Gamma_{\text{R}}\left(N_t - i + 1, \frac{N_t\gamma_{th}}{\rho_i}\right) \quad . \quad (12)$$

The following shows the analysis of the link diversity order.

Theorem 1: The diversity order of the ith link for an N_t-link ZF-DPC MIMO broadcast system without scheduling and ordering is $N_t - i + 1$.

Proof: To ease analysis, we define a function $Z_i(s)$ as

$$Z_i(s) = \frac{\log\left(1 - \Gamma_{\text{R}}(N_t - i + 1, N_t s\gamma_{th})\right)}{\log s} = \frac{\text{Num}_i(s)}{\text{Den}_i(s)} \quad , \quad (13)$$

where $s = \rho_i^{-1}$. When $\rho_i \to \infty$, $s \to 0$. Thus, we can have

$$\begin{aligned} D^i_{\text{order}} &= -\lim_{\rho_i \to \infty} \frac{\log P^i_{\text{out}}(\rho_i)}{\log \rho_i} \\ &= \lim_{s \to 0} Z_i(s) \quad . \end{aligned} \quad (14)$$

Note that $\lim_{s\to 0}\text{Num}_i(s) = -\infty$ and $\lim_{s\to 0}\text{Den}_i(s) = -\infty$ and according to the L'Hôpital's rule, we can obtain

$$\begin{aligned} D^i_{\text{order}} &= \lim_{s \to 0} \frac{\text{Num}'_i(s)}{\text{Den}'_i(s)} \\ &= \lim_{s \to 0} \left[\frac{(N_t s\gamma_{th})^{N_t - i + 1}\, e^{-N_t s\gamma_{th}}}{\Upsilon(N_t - i + 1, N_t s\gamma_{th})}\right] \quad , \end{aligned} \quad (15)$$

where $\Upsilon(a, x) = \int_0^x t^{a-1}e^{-t}dt$ is the lower incomplete gamma function. With the property $\Upsilon(a, x)/x^a \to 1/a$ as $x \to 0$, we can obtain the diversity orider $D^i_{\text{order}} = N_t - i + 1$ for an N_t-link ZF-DPC MIMO broadcast system without scheduling and ordering.

□

This theorem provides a surprising result that the ZF-DPC MIMO broadcast system can support extra diversity gains for $N_t - i$ links instead of traditional diversity order of one in the spatial multiplexing based MIMO systems. For example, the link diversity orders are respectively order three for $i = 1$, order two for $i = 2$, and order three for $i = 3$ as $N_t = 3$. The first two links obtain extra diversity gains under the ZF-DPC MIMO broadcast transmissions.

To derive cell coverage $R^i_{\mathtt{ZFDPC}}$ from (12), we first introduce the inverse of the regularized incomplete gamma function as follows:

$$x = \Gamma_{\text{R}}(a, z) \Rightarrow z = \Gamma_{\text{R}}^{-1}(a, x) \quad . \quad (16)$$

By substituting (4) and (16) into (12), the link coverage can be written as

$$R^i_{\mathtt{ZFDPC}} = \left[\frac{P_T 10^{g_0/10}}{N_t\gamma_{th}\sigma^2}\ \Gamma_{\text{R}}^{-1}\left(N_t - i + 1, 1 - P^i_{\text{out}}\right)\right]^{\frac{1}{\mu}} \quad , \quad i = 1, \ldots, N_t \quad . \quad (17)$$

B. ZFB without Scheduling

Alternately, we analyze the coverage performance of the ZFB MIMO broadcast system without user selection. In this case, all the elements in each channel vector are Rayleigh faded. Due to the same statistics, we can view the system as an point-to-point $N_t \times N_t$ (single user) MIMO system with a ZF receiver. The PDFs of the effective channel gain $\{b_i\}_{i=1}^{N_t}$ can be obtained through the PDFs of the ZF receiver's substream SNRs. According to [32], the distribution of the substream SNRs $\{\gamma_i\}_{i=1}^{N_t}$ for an $N_t \times N_r$ MIMO system with ZF receiver under equal power allocation are identically distributed $\mathcal{X}^2_{2(N_r - N_t + 1)}$. In the case of $N_t = N_r$, the PDF of unordered $\{b_i\}_{i=1}^{N_t}$ can be obtained from (9) by letting $i = N_t$, which result in exponentially distributed random variable with parameter one.

Therefore, the link outage probability and the coverage performance of the ZFB MIMO broadcast system can be obtained from (12) and (17) with $i = N_t$. Clearly, under the same link outage requirement, all ZFB substream links equal the ZF-DPC's weakest link and has the diversity order of one.

Corollary: The link diversity order of the ZFB MIMO broadcast system without scheduling is one.

V. Analysis of MIMO Broadcast Systems with Scheduling

A. ZF-DPC with Greedy Scheduling

Now we consider the effects of multiuser scheduling in the MIMO broadcast systems. We focus on the strongest stream link which has the largest radius to determine the cell range. In [33], the authors proposed a greedy scheduling algorithm to *select* N_t users out of K users to form $\mathbf{H}(\mathcal{S})$ and *ordering* those selected channel row vectors in the Gram-Schmidt orthogonalization to maximize the system throughput.

The strongest link can be determined by the first selected user's channel row vector $\mathbf{h}_k \in \mathcal{C}^{1\times N_t}$ for $k = 1, \ldots, K$. According to the greedy selection algorithm, the selected user k^* is

$$k^* = \arg\max_{k\in\{1,\ldots,K\}} d_{1,k} \quad , \quad (18)$$

where $d_{1,k} = \mathbf{h}_k\mathbf{h}_k^*$. Note that $d_{1,k}$ is the sum of N_t squared magnitudes of circularly symmetric, zero-mean, unit-variance complex Gaussian random variables. Therefore, $d_{1,k} \sim \mathcal{X}^2_{2N_t}$ with PDF $f_{d_{1,k}}(z) = z^{N_t - 1}e^{-z}/(N_t - 1)!$. The effective channel gain of the strongest link for the greedy scheduling algorithm is $\tilde{d}_1 = d_{1,k^*}$ of which PDF can be obtained by the order statistics analysis as follows:

$$f_{\tilde{d}_1}(z) = K[F_{d_{1,k}}(z)]^{K-1} f_{d_{1,k}}(z) \quad . \quad (19)$$

Hence the link outage probability is

$$\begin{aligned} P^1_{\text{out}} &= F_{\tilde{\gamma}_1}(\gamma_{th}) = F_{\tilde{d}_1}\left(\frac{N_t\gamma_{th}}{\rho_1}\right) \\ &= \left(1 - \Gamma_{\text{R}}(N_t, \frac{N_t\gamma_{th}}{\rho_1})\right)^K \quad . \end{aligned} \quad (20)$$

The following theorem gives the diversity order of the strongest link in the ZF-DPC MIMO broadcast system.

Theorem 2: The diversity order of the strongest link for the N_t-link ZF-DPC broadcast system with K-user greedy scheduling is KN_t.

Proof: Similar to the proof of Theorem 1, we define $\tilde{Z}_1(s)$ as

$$\tilde{Z}_1(s) = \frac{\log\left([1-\Gamma_{\mathrm{R}}(N_t, N_t s\gamma_{th})]^K\right)}{\log s} = \frac{\tilde{\mathrm{Num}}_1(s)}{\tilde{\mathrm{Den}}_1(s)} , \quad (21)$$

where $s = \rho_1^{-1}$. Thus, the diversity order is

$$\begin{aligned} \tilde{D}^1_{\mathrm{order}} &= -\lim_{\rho_1\to\infty} \frac{\log P^1_{\mathrm{out}}(\rho_1)}{\log \rho_1} \\ &= \lim_{s\to 0} \tilde{Z}_1(s) \\ &\overset{(a)}{=} \lim_{s\to 0} \frac{\tilde{\mathrm{Num}}_1'(s)}{\tilde{\mathrm{Den}}_1'(s)} \\ &= \lim_{s\to 0}\left[\frac{K(N_t s\gamma_{th})^{N_t} e^{-N_t s\gamma_{th}}}{\Upsilon(N_t, N_t s\gamma_{th})}\right] \\ &\overset{(b)}{=} KN_t , \end{aligned} \quad (22)$$

where (a) follows L'Hôpital's rule with $\lim_{s\to 0}\tilde{\mathrm{Num}}_1(s) = -\infty$ and $\lim_{s\to 0}\tilde{\mathrm{Den}}_1(s) = -\infty$ and (b) comes from the property $\Upsilon(a,x)/x^a \to 1/a$ as $x \to 0$.

□

To derive the link coverage $\tilde{R}^1_{\mathrm{ZFDPC}}$ of the strongest link from (20), we use the inverse of the regularized incomplete gamma function to obtain

$$\tilde{R}^1_{\mathrm{ZFDPC}} = \left[\frac{P_T 10^{g_0/10}}{N_t\gamma_{th}\sigma^2}\, \Gamma_{\mathrm{R}}^{-1}\left(N_t, 1-\sqrt[K]{P^1_{\mathrm{out}}}\right)\right]^{\frac{1}{\mu}} . \quad (23)$$

1) Effect of user ordering: Even with random user selection, the ZF-DPC MIMO broadcast system can still take advantage of users ordering. This case is similar to the multiuser MIMO broadcast system with $K = N_t$ users. The benefit of pure user ordering (not combined with users selection) will be shown in the section of numerical results.

2) Soft coverage extension by scheduling: To examine the benefits of multiuser scheduling, we define the *coverage extension ratio* η^1_{ZFDPC} as

$$\eta^1_{\mathrm{ZFDPC}} = \frac{\tilde{R}^1_{\mathrm{ZFDPC}}}{R^1_{\mathrm{ZFDPC}}} = \left[\frac{\Gamma_{\mathrm{R}}^{-1}\left(N_t, 1-\sqrt[K]{P^1_{\mathrm{out}}}\right)}{\Gamma_{\mathrm{R}}^{-1}(N_t, 1-P^1_{\mathrm{out}})}\right]^{\frac{1}{\mu}} , \quad (24)$$

where η^1_{ZFDPC} is a function of $\{N_t, K, P^1_{\mathrm{out}}, \mu\}$ and can be used to examine how N_t and K affect the reliable coverage range of the MIMO broadcast system. Here we take the strongest link of ZF-DPC MIMO broadcast system ($i = 1$) as an example to address the coverage extension issue. We will show that the reliable coverage increases as the number of users increases and decreases as the number of antennas increases in the numerical results.

For the other links ($i = 2, \ldots, N_t$) of ZF-DPC MIMO broadcast system with greedy scheduling, per link analysis is more difficult than no scheduling case. Although, the equation (46) in [2] provides the PDF of effective channel gains of ZF-DPC with greedy scheduling for $i = 2, \ldots, N_t$, this formula is complicated and intractable to derive its closed-form expression for link outage and coverage of ZF-DPC MIMO broadcast system with greedy scheduling so that the exact coverage extension gain can not be found easily.

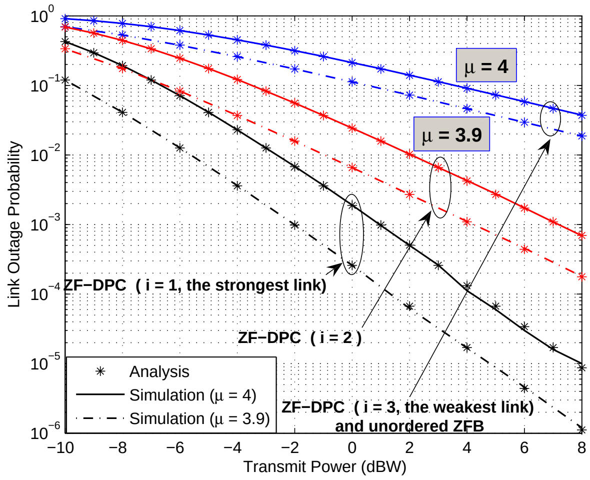

Fig. 2. Link outage probability performance against the transmit power P_T when path loss exponent $\mu = 3.9$ and 4 for both the ZF-DPC and ZFB MIMO broadcast systems, where $N_t = 3$, $\sigma^2 = -103$ dBm, $R = 1$ km and $\gamma_{th} = 2$ dB.

B. ZFB with Scheduling

For the ZFB MIMO broadcast system with scheduling, some suboptimal user selection algorithms [2] [3] were proposed to reduce the complexity of the exhaustive search. However, it is difficult to find the exact per link closed-form expression for the ZFB MIMO broadcast system with scheduling. To compare with the ZF-DPC MIMO broadcast system, we will show the simulation results of the ZFB MIMO broadcast system based on exhaustive search in Section VI.

VI. Numerical Results

In this section, we illustrate the achievable link outage and link coverage performances of both the ZF-DPC and ZFB MIMO broadcast systems. Assume that the predetermined value $\gamma_{th} = 2$ dB, $\sigma^2 = -103$ dBm, $g_0 = -32$ dB, $\mu = 4$ and $N_t = 3$.

Figure 2 shows the simulative and analytical link outage performances of both the ZF-DPC and ZFB MIMO broadcast systems without scheduling when user terminals are at the distance of $R = 1$ km from BS and path loss exponents $\mu = 3.9$ or 4. Clearly, the link outage probability becomes higher for a larger path loss exponent. Note that $\mu = 2$ is for free space, and $\mu = 3.5 \backsim 4$ is for two-path model of an urban radio channel. The diversity orders of different links match our analytical results in Theorem 1.

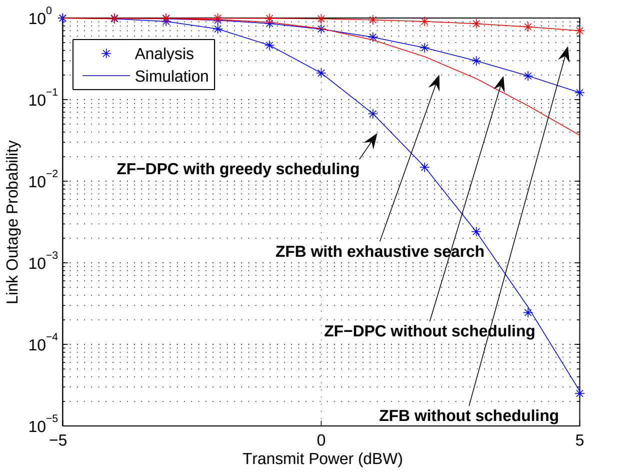

Fig. 3. Link outage probability performance against the transmit power P_T for the strongest links of both the ZF-DPC and ZFB MIMO broadcast systems with and without scheduling when $N_t = 3$, $\sigma^2 = -103$ dBm, $\mu = 4$, $R = 2$ km, $K = 5$ and $\gamma_{th} = 2$ dB.

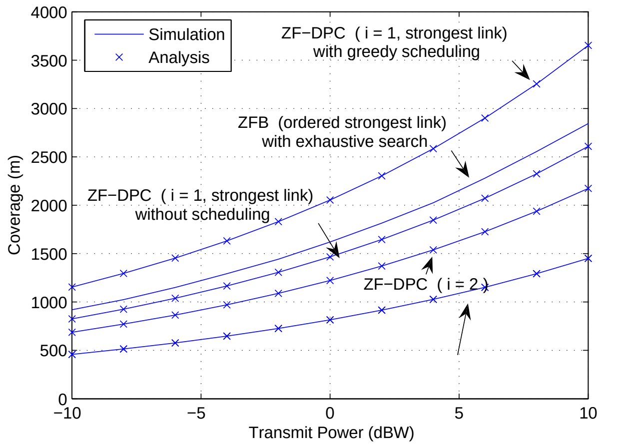

Fig. 4. Link coverage performance against the transmit power P_T for different stream links of both the ZF-DPC and ZFB MIMO broadcast systems when $N_t = 3$, $\sigma^2 = -103$ dBm, $\mu = 4$, $P_{out} = 0.1$, $K = 5$ and $\gamma_{th} = 2$ dB.

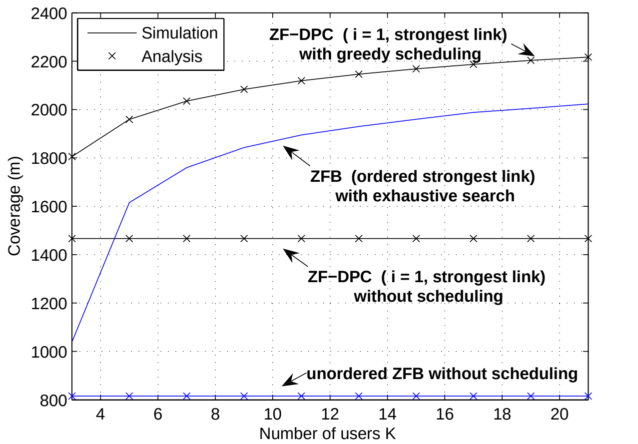

Fig. 5. Link coverage performance against the number of users K for different stream links of both the ZF-DPC and ZFB MIMO broadcast systems when $N_t = 3$, $\sigma^2 = -103$ dBm, $\mu = 4$, $P_{out} = 0.1$, $P_T = 0$ dBW and $\gamma_{th} = 2$ dB.

For example, the strongest link $i = 1$ has the diversity order of three, the link $i = 2$ has the diversity order of two, but the weakest link $i = 3$ has only the diversity order of one. Clearly, the ZF-DPC MIMO broadcast system can support extra diversity gains for $N_t - 1$ links instead of traditional diversity order of one in a spatial multiplexing based MIMO system. However, the broadcast system with ZF precoding has merely diversity order of one.

Figure 3 shows the link outage for the strongest links of both the ZF-DPC and ZFB MIMO broadcast systems with and without scheduling for five users ($K = 5$) at the distance of $R = 2$ km from BS. In the figure, it is shown that the multiuser diversity gain is still significant even if the degree of freedom is merely $K = 5$. From this figure, the curve of ZF-DPC with greedy scheduling tends to have the diversity of order $KN_t = 15$. As a result, the deficient diversity of the spatial multiplexing based MIMO broadcast system can be compensated by taking advantage of multiuser scheduling.

Figure 4 shows the corresponding link coverage performance of Fig. 3 in which we set the link reliability as 0.9 under $\gamma_{th} = 2$ dB. Clearly, coverage is extended in both the ZF-DPC and ZFB MIMO broadcast systems with scheduling even with $K = 5$. For example, it can only maintain 90% link reliability as far as about 1.6 km radius without scheduling, but can extend to 2.1 km with scheduling for the ZF-DPC MIMO broadcast systems when $P_T = 0$ dBW, i.e. the achievable coverage increases 31.25% by the help of multiuser scheduling.

Figure 5 shows the coverage improvement for a different numbers of users in the ZF-DPC and ZFB MIMO broadcast systems when $P_T = 0$ dBW. The benefit of user ordering can be clearly observed from the coverage enhancement of ZF-DPC's strongest link at $K = N_t = 3$. Specifically, the cell radius is improved from 1.46 km to 1.8 km.

Figure 6 shows the coverage extension gain with different N_t and K according to (24). One can see that the benefit of multiuser scheduling is significant as K increases. However, the multiuser scheduling gain will reduce as more antennas are employed at the BS, i.e., for a larger N_t. From the above numerical results, we know that **soft coverage** enhancement can be achieved by applying multiuser scheduling techniques without increasing transmission power. That is, link quality is improved by multiuser diversity so that the reliable coverage can be extended. As the number of antennas equipped at a base station increases, transmit power allocated to each link will decrease under the same transmit power constraint. As a result, it will be hard for the MIMO broadcast system to maintain the same reliable coverage with a predetermined SNR requirement.

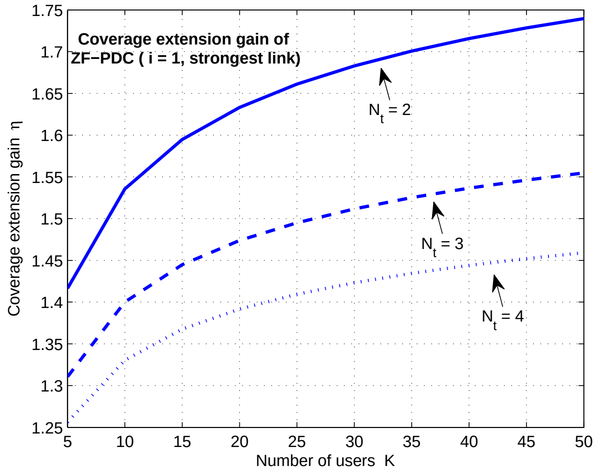

Fig. 6. Coverage extension gain against the number of users K for the strongest link of the ZF-DPC MIMO broadcast system with $N_t = 2, 3$, and 4.

VII. Conclusion

In this paper, we have analyzed the link outage, diversity order, and link coverage performance for the multiuser MIMO broadcast systems. We derive analytical closed-forms of the link outage probability, diversity order and reliable link coverage for both the ZF-DPC and ZFB MIMO broadcast systems. We define the coverage extension ratio to demonstrate how multiuser scheduling can improve the reliable coverage of the MIMO broadcast system without increasing BS transmission power. From our analysis, the reliable coverage can be extended significantly as the number of users increases, but the performance gain due to multiuser scheduling is reduced as the antennas installed at a BS increase.

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